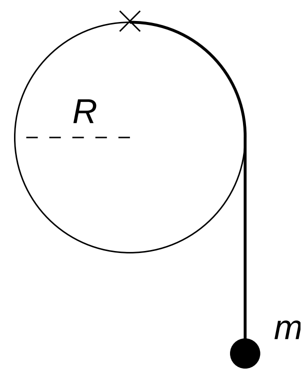
Classical Physics **Name** ____________________

Ph.D. Qualifying Exam, Spring 2006

INSTRUCTIONS: There are nine problems on this exam: Four in section A, three in section B and two in section C. You must solve SIX problem with at least two from sections A and B and at least one from section C. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

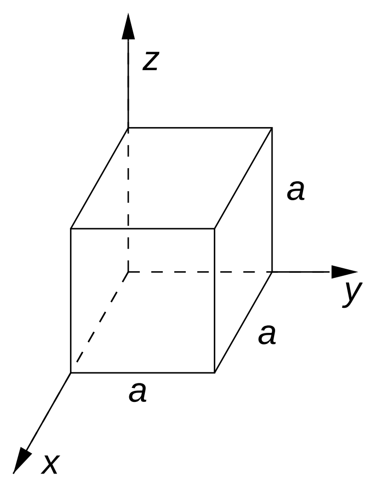
Problem A1: Consider a particle of mass m constrained to move on the surface of cone whose apex is pointing down. The particle is subject only to gravity and the force of constraint. Determine the differential equation of motion. What, if any, quantities are conserved?

Problem A2: A pendulum is constructed by attaching a mass m to an extension-less string of length L with the other end of the string connected to the uppermost point on a vertical disk of radius R ($R < L/\pi$). Obtain the pendulum's equation of motion and find the frequency of small oscillations using Lagrange's equations.



Problem A3: A particle is at rest at the top of a smooth hemisphere of radius R that is oriented with its flat side down and fixed in space. After a small disturbance, the particle slides down the hemisphere. At what point does the particle lose contact with the sphere surface?

Problem A4: A cube with sides a, mass m, and uniform density is located in a Cartesian coordinate system as shown.



a) Calculate the inertia tensor I:

$$I_{ij} = \int dm(r^2\delta_{ij} - x_i x_j).$$

Be sure to take advantage of any symmetries.

b) Find the eigenvalues of I, i.e., the rotational inertias about the principal axes, and determine the degree of degeneracy.

c) For the lowest eigenvalue find the direction of the principal axis.

Problem B1: A sphere of radius a with its center located at the origin has a surface charge density given in spherical coordinates by $\sigma = \sigma_0 \cos\theta$, where σ_0 is a constant. Find the monopole moment, the dipole moment, and all components of the quadrupole tensor. Express the potential at a field point outside the sphere in terms of the spherical coordinates of the point.

Problem B2: A parallel plate capacitor consists of two circular plates each of area S with vacuum between them. The capacitor is connected to a battery of constant emf, E. The plates are then slowly oscillated so they remain parallel but the separation d between the plates is given by $d = d_0 + d_1 \sin\omega t$. Find the magnetic field, $\vec{H}$, between the plates produced by the displacement current. Similarly, find $\vec{H}$ if the fully-charged capacitor is first disconnected from the battery before the plates are oscillated.

Problem B3: A cubic box with sides of length a is constructed of conducting planes that are insulated from each other. In a coordinate system with the origin at one corner of the box, the four sides at $x = 0$, $x = a$, $y = 0$, and $y = a$ are grounded ($V = 0$) and the two sides at $z = 0$ and $z = a$ are at a potential $V_0 \neq 0$.

a) Write down a general solution to Laplace's equation for the potential inside the box that satisfies the boundary conditions on the four sides of the box that are grounded. Your result should consist of a linear combination of products of function of the three variables x, y, z. Hint: to satisfy the boundary conditions, you will need to use the oscillatory functions in the x and y variables.

b) Use the remaining boundary conditions at $z = 0$ and $z = a$ to determine the coefficients in the result for part a). Your final result for the potential should consist of a series of terms with coefficients expressed in terms of V_0 and a.

Problem C1: The behavior of a four stroke gasoline engine can be approximated by the so-called Otto cycle: The process is as follows: 1) Isobaric intake (from $V = 0$) at atmospheric pressure up the volume V_1 (maximal volume) and temperature T_1. 2) Adiabatic compression to volume V_2 and temperature T_2. 3) Isochoric increase of temperature during ignition to T_3. 4) Adiabatic expansion to volume V_1 and temperature T_4. 5) Isochoric decrease of temperature to T_1. 6) Isobaric exhaust to $V = 0$ at atmospheric pressure.

Sketch the process in a PV diagram. Assume the working substance is an ideal gas with $\gamma = C_P/C_V$ and calculate the efficiency of this process. Express the result in terms of the temperatures T_1, T_2, T_3, and T_4. If the compression ratio is given by $r + V_1/V_2$, express the efficiency in terms of r and γ.

Problem C2: The equation of state for radiant energy in equilibrium with the temperature of the walls of a cavity of volume V is $P = aT^4/3$, where a is a constant. The internal energy is $U = aT^4V$.

a) Calculate the amount of heat supplied in an isothermal doubling of the volume of the cavity.

b) Show that an adiabatic process is characterized by $VT^3 = const.$ Hint: use the first law and $U = U(V,T)$.

Classical Physics **Name** ______________________

Ph.D. Qualifying Exam, Spring 2007

INSTRUCTIONS: There are nine problems on this exam: Four in section A, three in section B and two in section C. You must solve SIX problem with at least two from sections A and B and at least one from section C. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

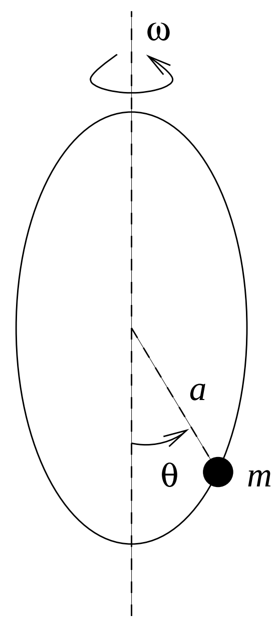
Problem A1: A uniform rod of length L and mass m stands vertically on a frictionless horizontal plane. It is then slightly perturbed from this position. Determine the angular speed, $\dot{\theta}$, when the rod has turned through an angle θ and when it hits the horizontal plane.

Problem A2: A bead of mass m moves without friction on a circular wire hoop of radius a. The hoop rotates with a constant angular velocity ω about the vertical diameter.

a) Find the equation of motion for the bead.

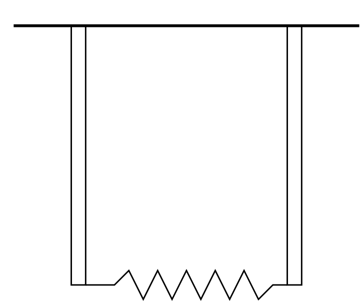
b) What are the constants of motion?

c) Show that if ω is greater than some critical value ω_0, there can be a solution in which the bead remains stationary on the hoop at a point other than the bottom, but that if $\omega < \omega_0$, the only stationary point for the bead is at the bottom. Find ω_0.



Problem A3: A heavy loop of rope of length $2\pi a$ just fits around a frictionless cylinder of radius a. If the axis of the cylinder is horizontal, show that the tension at the highest point is 3 times the tension at the lowest point.

Problem A4: Two identical thin rods are suspended at one end and connected by a massless spring at the other end which is unstretched when the rods hang vertically. If each rod has length L and mass m, and k is the spring constant, find the normal modes and the oscillation frequencies. Assume small displacements from the vertical direction and neglect friction.



Problem B1: A circular loop of radius R carries a current I.

a) Show that the magnetic field, B, at any point on the axis of the loop points along the axis and derive an expression for the magnetic field strength in terms of R and I, and the distance z from the center of the loop.

b) Now consider two such identical loops parallel to each other with their centers along the same axis separated by a distance b with each carrying current I in the same direction. Choose the origin of the z axis midway between the two loops so that the centers are at $z = -b/2$ and $z = b/2$. Now expand the total magnetic field in powers of z for points near $z = 0$. Show that the lowest order term (zeroth order in z) is

$$B_0 = \mu_0 I R^2 / d^3$$

with $d^2 = R^2 + b^2/4$.

c) Show that the lowest order correction to the expression above has the form

$$B_2 = \alpha B_0 z^2,$$

and evaluate the constant α in terms of R, b, and d.

d) For the case of $b = R$ (Helmholtz coils), show that the lowest order correction to B_0 is of order z^4. You need not evaluate this term.

Problem B2: Start from the integral form of Faraday's law of induction,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A},$$

and derive its differential form:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}.$$

Hint: Use infinitesimal square loops in the three different planes.

Problem B3: A parallel plate capacitor is formed by a cylinder with non-conducting walls and a conducting bottom and a conducting, movable piston. The cylinder is airtight and kept at constant temperature. When the capacitor is uncharged, the separation of the plates is d_0 and the pressure inside the cylinder is P_0. When a potential difference of $\Delta\phi$ is applied between the plates, show that the fractional decrease, f, in plate separation ($f > 0$) is given by:

$$f(1-f) = \frac{\epsilon_0}{2P_0} \left(\frac{\Delta\phi}{d_0} \right)^2 .$$

Problem C1: Consider an ideal gas whose pressure and volume are given by $P(t) = 1 + 0.25\cos(2\pi t)$ and $V(t) = 1 + 0.25\sin(2\pi t)$. Draw a picture of state of the gas as a function of t in a $P - V$ plot. Find the work done by the gas in one cycle. Repeat the above for $P(t) = 1 + 0.25\cos(2\pi t)$ and $V(t) = 1 - 0.25\sin(2\pi t)$ and $P(t) = 1 + 0.25\cos(2\pi t)$ and $V(t) = 1 - 0.25\cos(2\pi t)$. Give a physical explanation for the results obtained for the work done in all three cases.

Problem C2: Given equation of state as $P(v - b) = RT$, show that the relation between P and v during an adiabatic process is $P(v-b)^\gamma = constant$, where γ is the ratio of C_p and C_v.

Classical Physics **Name** ______________________________

Ph.D. Qualifying Exam, Fall 2007

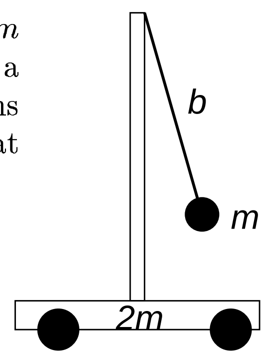
INSTRUCTIONS: There are nine problems on this exam: Three in section A, four in section B and two in section C. You must solve SIX problem with at least two from sections A and B and at least one from section C. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: A particle of mass m is launched upward from the ground with an initial velocity $\dot{z} = v_0$. The particle is subject to a retarding force of $\vec{F}_R = -k\vec{v}$, where k is a constant, in addition to gravity.

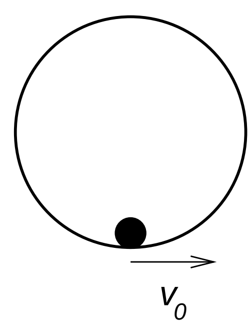
a) Determine the maximum height obtained by the particle.

b) As the particle returns to earth, find the maximum possible velocity of the particle assuming that it was launched with a velocity greater than this maximum return velocity.

Problem A2: A simple pendulum of length b and mass m is attached to a cart of mass $2m$ that is allowed to roll on a horizontal surface (see attached figure). Find the equations of motion. Assume that the system is frictionless and that the wheels of the cart are massless.



Problem A3: A particle of mass m is moving without friction inside of a vertical circular track of radius R. When it is at its lowest position, its speed is v_0. Use the Lagrange method to find the minimum value of v_0 for which the particle will go completely around the circle without losing contact with the track?



Problem B1: a) Two conducting spheres of radii a and b have their centers a distance c apart. If $c >> a$ and $c >> b$, show that the capacitance of the system is given by

$$C = 4\pi\epsilon_0 \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{c}\right)^{-1}.$$

b) Two infinitely long conducting cylinders of radii a and b are separated by a distance c. Again $c >> a$ and $c >> b$. Find an approximate expression for the capacitance of a length L of this system.

Problem B2: A spherical cavity of radius a is within a large, grounded conductor. A charge q is placed within the cavity at a distance b from the center.

a) Find the potential at all points within the cavity. Use spherical coordinates with origin at the center and z axis passing through q. *Hint:* Place an image charge q' at $z = d$, where $d > a$.

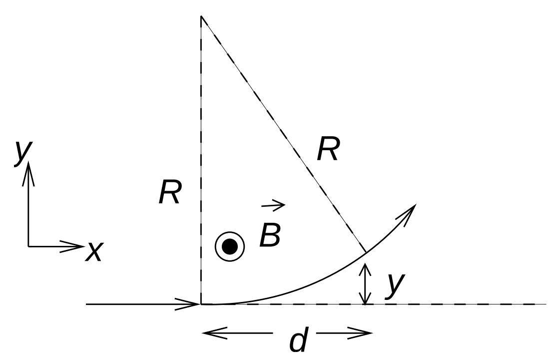
b) Find the electric field, $\vec{E}$ at all points within the cavity. What is $\vec{E}$ at the center of the cavity?

c)What is the surface charge density induced on the wall of the cavity?

d) What is the total induced charge on the wall?

Problem B3: An electromagnetic "eddy current" brake consists of a disk of conductivity σ and thickness d rotating about an axis through its center that is perpendicular to the disk. A uniform magnetic field, $\vec{B}$, is applied perpendicular to the disk over a small area a^2 located at a distance ρ from the axis. Show that the torque tending to slow down the disk at the instant its angular speed is ω is given approximately by $\tau \approx \sigma\omega B^2\rho^2a^2d$.

Problem B4: The charge and mass of the electron can be determined in a pair of experiments performed in succession. In the first experiment, an electron with known kinetic energy K enters a region with a uniform magnetic field oriented perpendicularly to the electron velocity. The magnetic field causes the electron to move through an arc of radius R such that over a horizontal distance d, it is deflected vertically a distance y. In the second experiment, the original magnetic field is supplemented by a unifom electric field chosen so that the electron now passes through the region of both fields without being deflected.



a) Determine the radius R in terms of the distances d and y.

b) Noting that the centripetal force acting on the electron as it moves through the circular arc in the first experiment is due to the magnetic field, derive an expression for R in terms of the magnetic field strength B, the charge and mass of the electron, and the kinetic energy K.

c) Determine the direction of the electric field (relative to the coordinate system in the figure) in the second experiment. Derive an expression for the electron mass in terms of K, B, and the electric field strength E.

d) Use the results of parts a), b), and c) above to determine the electron charge magnitude in terms of the distances d and y, the field strengths E and B, and the kinetic energy K.

Problem C1: Compute the efficient of a heat engine with the following cycle: Compression at constant pressure P from state A to state B; Adiabatic compression from state B to state C at pressure rP; Expansion at constant pressure rP from state C to state D; Adiabatic expansion from state D to state A.

Problem C2: Assume molar latent heat L of evaporation is constant and the gas phase can be treated as ideal gas, also assume the specific volume of liquid phase is negligible.

a) Calculate the saturated gas pressure as a function of temperature.

b) Calculate the expansion coefficient $\frac{1}{v}\frac{dv}{dT}$ along the coexistence curve as function of temperature.

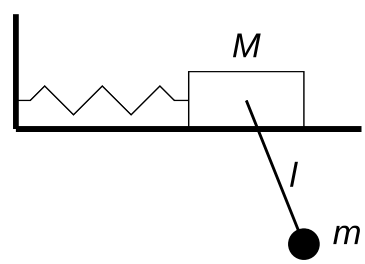
Classical Physics **Name** ______________________________

Ph.D. Qualifying Exam, Fall 2008

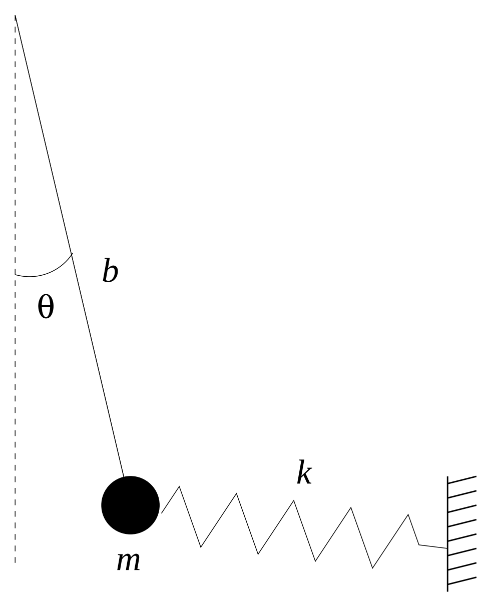
INSTRUCTIONS: There are nine problems on this exam: Three in section A, four in section B and two in section C. You must solve SIX problems with at least two from sections A and B and at least one from section C. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: A projectile is fired at an angle of 45 degrees with an initial kinetic energy E. At the top of its trajectory, the projectile explodes with additional energy E into two fragments. One fragment of mass m_1 travels straight down. Immediately after the explosion, what is the velocity (magnitude and direction) of the second fragment m_2 and what is the velocity of the first fragment? What is the ratio m_1/m_2 when m_1 is a maximum?

Problem A2: A block of mass M is attached to a spring with the spring constant k. A pendulum bob of mass m is hung from from the block with a weightless rod of length l. Find the frequencies of small amplitude oscillations (Neglect friction).



Problem A3: A mass m is attached to the bottom of a string of length b. A spring of force constant k is attached to the mass and a nearby wall, such that the spring is free to rotate about the two end points. The unstretched spring reaches from the mass to the wall when the angle $\theta = 0$. The mass is then pulled to an initial angle θ_0 and released.



a) Write down the Lagrangian of the system, using only the variable θ and whatever constants are needed. [Take the potential energy to be $U = 0$ at the bottom of the string.]

b) For small angles, find the frequency of oscillation.

Problem B1: An infinitely long, conducting cylinder of radius a is co-axial with an infinitely long, hollow, conducting cylinder of inner radius b and outer radius c. The space between them is filled with a dielectric for which the dielectric constant κ is given in cylindrical coordinates by $\kappa = \alpha\rho^n$ where α and n are constants. There is free charge λ_f per unit length on the inner cylinder. Find $\vec{D}$, $\vec{E}$, and the volume density ρ_b of bound charge everywhere between the conductors. Under what circumstances will E be constant, and what then will be the corresponding values of $\vec{D}$ and ρ_b?

Problem B2: A plane electromagnetic wave traveling in a vacuum is given by $\vec{E} = E_0 e^{i(kz-wt)}\hat{y}$, where E_0 is real. A circular loop of radius a, N turns, and resistance R is located with its center at the origin. The loop is oriented so that its diameter lies along the z axis and the plane of the loop makes an angle θ with the y axis.

a) Find the emf induced in the loop as a function of time. Assume that $a << \lambda$.

b) Find the time-averaged rate of heating in the loop.

Problem B3: A parallel plate, vacuum capacitor with circular plates, each of area S, is connected to a battery of constant emf, ε. The plates are then slowly oscillated so they remain parallel, but the gap d between them is given by $d = d_0 + d_1 \sin\omega t$.

a) Find the magnetic field, $\vec{H}$, between the plates produced by the displacement current.

b) Similarly find $\vec{H}$ if the charged capacitor is first disconnected from the battery and then the plates are oscillated as before.

Problem B4: Within the region $0 \leq x \leq a$ of the vacuum, the electric and magnetic fields are given by

$$\begin{aligned} E_y &= -B_0(\omega a/\pi)\sin(\pi x/a)\sin(kz-\omega t) \\ B_x &= B_0(ka/\pi)\sin(\pi x/a)\sin(kz-\omega t) \\ B_z &= B_0\cos(\pi x/a)\cos(kz-\omega t) \end{aligned}$$

with $(\omega/c)^2 = k^2 + (\pi/a)^2$, where $c^2 = (1/\mu_0\epsilon_0)$. All other components of the fields are zero. B_0, k, a, and ω are all constants.

a) Show that Maxwell's equations are satisfied by these fields in the absence of volume currents and charges.

b) Calculate the Poynting vector within the region. Show that the time-averaged Poynting vector has only a z-component and then determine the time-averaged power transmitted across a rectangular surface with boundaries at $x = 0$, $x = a$, $y = 0$ and $y = b$ which is oriented perpendicularly to the z-axis.

c) Suppose that the region of interest is bounded by perfectly conducting planes at $x = 0$ and $x = a$, which extend infinitely in the y and z directions. Determine the surface charge density and the surface current density at arbitrary points on both surfaces as functions of position on the surfaces. Assume that the fields vanish at $x < 0$ and at $x > a$.

Problem C1: A particle has the energy level scheme:

$$
\begin{array}{ll}
\epsilon_3 & - \; - \; - \\
\epsilon_2 & \;\; - \; - \\
\epsilon_1 & \;\;\;\; -
\end{array}
$$

meaning that there is two-fold degeneracy in level ϵ_2 and three-fold degeneracy in level ϵ_3.

a) In the limit of $T \to 0$, what is the average energy, $\bar{E}$, and free energy, F, of the particle, where $F = -kT \ln Z$?

b) In the limit of $T \to \infty$, what is the the average energy, $\bar{E}$, and free energy, F, of the particle?

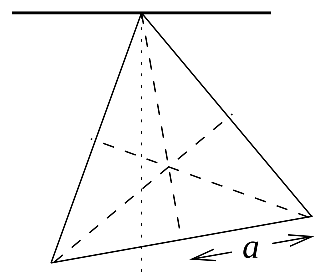
Problem C2: Compute the efficiency of a heat engine with the following cycle: (1) Isothermal expansion from state A to state B at T_h; (2) Cooling at constant volume from state B at T_h to state C at T_c; (3) Adiabatic compression from state C to state A. Compare the efficiency to an heat engine operating on a Carnot cycle ($\epsilon = 1 - T_C/T_H$).

Classical Physics **Name** ____________________________

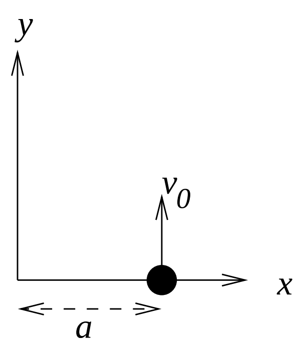
Ph.D. Qualifying Exam, Fall 2009

INSTRUCTIONS: There are nine problems on this exam: Four in section A, three in section B, and two in sectionn C. You must solve a total of SIX problems with at least two from sections A and B and one from section C. You must also solve the two problems marked with *. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

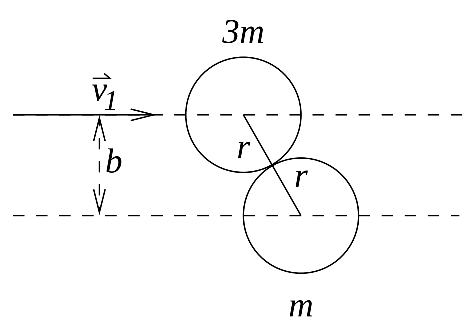
Problem A1: A uniform plate of mass m in the form of an equilateral triangle of side $2a$ is suspended from a horizontal axis through one of its vertices, perpendicular to its plane. Determine the frequency of small oscillation about the vertical equilibrium position for motion in the plane of the triangle.



Problem A2*: A particle of mass m moves in an attractive central field of force $-mn^2r$, which is proportional to the distance from the center of the force. The motion starts at $t = 0$ with the particle at $x = a$, $y = 0$ and $v = v_0$, where v_0 is perpendicular to the radius vector. Find the Lagrangian for the particle and determine the position of the particle at later times. Also find the equation for the orbit; that is, use $x^2 + y^2 = r^2$ to get a single *simplified* equation describing the orbital motion.



Problem A3: Two perfectly smooth and elastic disks of radius r, masses $3m$ and m slide on a smooth horizontal surface. The $3m$ disk strikes the m disk, which is initially at rest, with a speed of v_1. The impact parameter b is defined in the diagram. For what value of b are the directions of motion of the disks after the collision at 45° with respect to each other?



Problem A4: A particle of mass m moves on the surface of a circular cone of half angle α with its axis in the vertical direction. Gravity acts downward.

a) Write the equations of motion and find the conditions for motion of the particle to remain at a constant height h above the cone's vertex.

b) Find the frequency of small oscillations about this horizontal trajectory.

Problem B1: A solenoid consists of a hollow cylinder of length L and cross sectional-radius a tightly wound with an insulating wire so that the wire makes N circular loops around the cylinder. The wire carries a current I.

a) For arbitrary values of L and a (i.e. do not assume $L >> a$), determine the magnetic field within the solenoid at an arbitrary point on the axis of the cylinder. Hint: The necessary integral over z is most easily accomplished by making a trigonometric substitution.

b) Evaluate the result of part a) at the center of the solenoid and at the ends along the axis and thereby show that in the limit as $L >> a$, the field strength at the center is twice that at the ends.

Problem B2: A dielectric sphere with a radius a is uniformly polarized in the positive z direction, $\vec{P} = P\hat{z}$. It is rotated with constant angular velocity $\vec{\omega} = \omega\hat{z}$ about a diameter coinciding with the z axis. Assuming that the polarization is not affected by the rotation, find the magnetic field $\vec{B}$ at the point where the axis of rotation intersects the surface of the sphere, i.e. at $z = a$. What is $\vec{B}$ at the center of the sphere?

Problem B3*: A coaxial cable consists of a very long cylindrical wire of radius a surrounded concentrically by a very long cylindrical shell of inside radius b and outside radius c $(a < b < c)$. The inside wire carries a current I along its length, uniformly distributed across its cross section. The outside shell carries a return current of the same magnitude in the opposite direction, also uniformly distributed across its cross section.

a) Use Ampere's law to determine the magnetic field at arbitrary points in the three regions $\rho < a$, $a < \rho < b$, and $b < \rho < c$, where ρ is the radial distance to the point from the common axis of the cylinders.

b) Calculate the magnetic energy per unit length of the cable and then use this to determine the inductance of the cable per unit length.

c) Assuming that Ohm's law is obeyed in both the inside wire and outer shell and that both conductors have the same electrical conductivity σ, determine the Poynting vector at an arbitrary point inside each conductor.

d) Show that the power per unit axial length that flows **radially** into the two conductors is equal to the power per unit length that is dissipated by the electrical resistance of the cable.

Problem C1: The Maxwell speed distribution is given by:

$$F(v)dv = 4\pi C e^{-\frac{1}{2}\beta m v^2} v^2 dv.$$

a) What is the most probably speed, v^*?

b) What fraction of the molecules in an ideal gas in equilibrium have speeds within $\pm 1\%$ of v^*?

Problem C2: An ideal gas goes through an expansion from V_i to V_f. The initial pressure is P_i. Compute the work done by the gas assuming the expansion was at constant pressure. Compute the work done by the gas assuming the expansion was at constant temperature. In which case does the gas do more work?

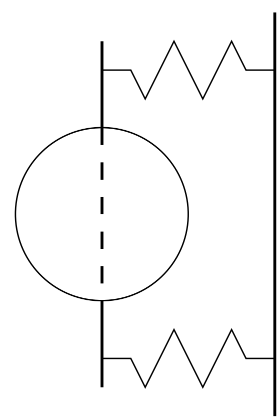
Classical Physics **Name** ______________________

Ph.D. Qualifying Exam, Fall 2010

INSTRUCTIONS: There are nine problems on this exam: Three in section A, four in section B and two in section C. You must solve SIX problem with at least two from sections A and B and at least one from section C. You must also solve the problems marked with *. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k. Assume that the spring stretches without bending and swings a vertical plane. Using the angular displacement of the mass from the vertical and the length that the spring has stretched from rest, find Lagrange's equations of motion.

Problem A2*: The figure shows the top view on a solid sphere with mass m and radius r that is resting on the floor. The sphere has a horizontal shaft passing through its center which is attached to a wall at both ends with two springs with force constant k. There is sufficient friction between the sphere and the floor such that the sphere will roll without slipping. When displacing the sphere from it's equilibrium position, what is the period of the resulting oscillation? Assume that the two springs are stretched equally. The moment of inertia for a solid sphere is $\frac{2}{5}mR^2$.



Problem A3: The engine of a car with mass m delivers a power P at full throttle. In addition to the resulting driving force, the car is also subject to a drag force that is proportional to the velocity of the car ($F_d = -bv$). Find $v(t)$ assuming a constant P. What is the limiting speed of the car as $t \to \infty$?

Problem B1: A uniformly charged cylindrical shell of inside radius a, outside radius b ($a < b$), and length L has a total charge q. The cylindrical shell spins with angular velocity ω about its vertical axis.

a) Choosing a coordinate system with the origin at the center of the bottom surface of the cylinder and with the z-axis along the cylinder axis (so the bottom and top surfaces are at $z = 0$ and $z = L$ respectively), determine the volume current density within the cylindrical shell.

b) Determine the magnetic dipole moment of the shell.

c) Using the result of part b), find an approximate expression for the magnetic field produced by the shell at an arbitrary distant point, i.e., for $r >> L$ and $r >> b$, where r is the distance to the point from the origin. Express your result in spherical coordinates, i.e., in terms of r and θ (the angle to the observation point relative to the cylinder axis).

d) What is the magnitude of the field determined in part c?

Problem B2*: The dipole moment of a rotating electric dipole can be expressed as a function of time by the expression

$$\vec{p}(t) = p_0 \hat{\rho}(t)$$

where the time dependent unit vector $\hat{\rho}(t)$ is given by

$$\hat{\rho}(t) = \hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)$$

and $\hat{x}$ and $\hat{y}$ are the unit vectors in the x and y directions. p_0 is the dipole moment strength and ω is the angular velocity of the dipole.

a) In the radiation zone, the electric and magnetic fields of the electromagnetic radiation emitted by a moving electric dipole are given by

$$\vec{E}(t) = \left(\frac{\mu_0}{4\pi r}\right)\left[\hat{r}\left(\hat{r}\cdot\frac{d^2\vec{p}}{dt^2}\right) - \frac{d^2\vec{p}}{dt^2}\right],$$

and

$$\vec{B}(t) = \frac{1}{c}\hat{r}\times\vec{E}(t)$$

where $\hat{r} = \hat{z}\cos\theta + (\hat{x}\cos\phi + \hat{y}\sin\phi)\sin\theta$ is the unit vector in the radial direction and the time variable is now the retarded time rather than the physical time. The angles θ and ϕ are the usual polar and azimuthal angles in the spherical coordinate system. From the general expression for $\vec{E}(t)$, show that the electric field associated with the rotating dipole in the radiation zone can be expressed in the form

$$E(t) = \frac{\mu_0\omega^2 p_0}{4\pi r}\left[\hat{\theta}\cos\theta\cos(\omega t - \phi) + \hat{\phi}\sin(\omega t - \phi)\right],$$

where the spherical unit vectors are given by

$$\hat{\theta} = -\hat{z}\sin\phi + (\hat{x}\cos\phi + \hat{y}\sin\phi)\cos\theta$$

and

$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

b) Using the results of part a, show that the time dependent Poynting vector associated with the rotating dipole in the radiation zone can be expressed as

$$\vec{S}(t) = \frac{|E(t)|^2}{\mu_0 c}\hat{r}.$$

Then use this expression to show that the angular distribution of the radiated power, averaged over the retarded time, is proportional to $(1+\cos^2\theta)$ and find the coefficient of the proportionality. Note that after averaging over time, your result should *not* depend on the time.

Problem B3: A very long wire oriented along the y-axis carries a current I in the positive y-direction. In the xy-plane on the positive x side of the wire is an equilateral triangle with sides of length a. One side of the triangle is parallel to the wire and a distance d from the wire. The triangle carries a current of the same magnitude I which is directed so that the current through the side parallel to the wire is in the same direction as the current through the wire. Find the magnetic force exerted by the wire on the loop.

Problem B4: A very long cylinder of radius a is oriented with its axis along the z-axis. The cylinder has a uniform permanent polarization of magnitude P which points in the positive x-direction. The cylinder has no free charge, either within its volume or on its surface.

a) By fitting boundary conditions to the appropriate solutions of Laplace's equation, determine the electric potentials at arbitrary points within the cylinder ($\rho < a$) and outside of it. Note that the general solution to Laplace's equation in cylindrical coordinates with no free charge and no z-dependence is given by the expression

$$\Phi(\rho,\phi) = V_0 + \sum \left([a_n\cos(n\phi) + b_n\sin(n\phi)]\,\rho^n + [c_n\cos(n\phi) + d_n\sin(n\phi)]\,\phi^{-n} \right)$$

where the ρ and ϕ are cylindrical radial and angular coordinates, and the summation index ranges from 1 to infinity.

b) Use the results of part a to show that the electric field at an arbitrary point *inside* the cylinder points in the opposite direction as that of the polarization. Find the electric field at an arbitrary point *outside* the cylinder.

Problem C1: Prove that for a system in which the coefficient of thermal expansion is given by $\alpha = 1/T$ then c_p is independent of the pressure, i.e. show that

$$\left(\frac{\partial c_p}{\partial P}\right)_T = 0,$$

with

$$c_p = \left(\frac{\partial Q}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P, \quad \text{and} \quad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Hint: You will need to use a Maxwell Relation. Also, since T and P are the independent parameters then $\partial T/\partial P = 0$, and the ordering $\partial^2[\ ]/(\partial P \partial T) \leftrightarrow \partial^2[\ ]/\partial T \partial P$ can be interchanged.

Problem C2: An ideal monoatomic gas starts at state $A = (P_1, V_1)$. It then goes to a state $B = (P_2, V_1)$ and then to state $C = (P_2, V_2)$. You are given that $P_2 > P_1$ and that $V_2 > V_1$. From state C it goes to state D via isothermal expansion and it returns to state A via adiabatic compression. Find the change in internal energy, heat input into the gas and work done by the gas for all four paths in its cycle.

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS
UNIVERSITY PARK, MIAMI, FL33199

Ph.D. Qualifying Exam, Fall 2011 – CLASSICAL PHYSICS

FRIDAY, AUGUST 19, 2011 1PM - 5PM

INSTRUCTIONS:

1. There are nine problems on this exam with
   - four in **MECHANICS**;
   - three in **ELECTROMAGNETISM**;
   - two in **THERMODYNAMICS/STATISTICAL PHYSICS**.
2. You must solve a total of **SIX** problems with
   - at least two from **MECHANICS**;
   - at least two from **ELECTROMAGNETISM**;
   - and at least one from **THERMODYNAMICS/STATISTICAL PHYSICS**.
3. You **MUST** solve the problems marked **REQUIRED**.
4. Do each problem on separate sheets of paper and turn in only those problems you want to have graded.
5. Write your name and problem number on each page.

MECHANICS

1. A block of mass m is initially at rest on a frictionless horizontal plane. Starting at time $t = 0$, the plane is raised with one end held fixed, so that the inclination angle of the plane increases at a constant rate (i.e., so that $\frac{d\theta}{dt} = \alpha$). This causes the block to slide down the inclined plane.

   (a) Find expressions for the kinetic energy and gravitational potential energy of the block at a particular moment as functions of the variables θ, the inclination angle of the block, and s, the distance of the block along the plane from the end touching the ground.

   (b) Using the Euler-Lagrange formalism, derive the equation of motion for the block.

   (c) Assuming that $s = L$ at time $t = 0$, solve the equation obtained in part (b) to determine s as a function of time.

2. A projectile is launched due north from a point in colatitude θ ($\theta = 0$ is the north pole) at an angle $\frac{\pi}{4}$ to the horizontal, and aimed at a target whose distance is y (small compared to Earth's radius R). Show that if no allowance is made for the effects of the Coriolis force, the projectile will miss its target by a distance

$$x = \omega\sqrt{\frac{2y^3}{g}}\left(\cos\theta - \frac{1}{3}\sin\theta\right) + O(\omega^2) \tag{1}$$

   where ω is the angular velocity due to the Earth's rotation.

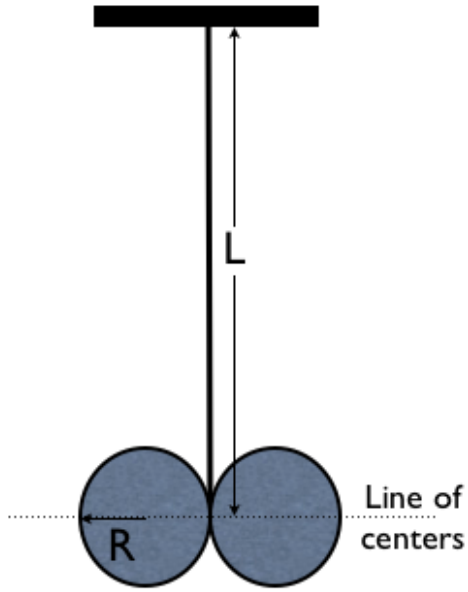


Figure 1: Figure for Problem 3 in Mechanics

3. A pendulum consists of a light rigid rod of length L with two identical solid spheres of radius R attached one on either side of its lower end (see Figure 1). Find the period of small oscillations (a) perpendicular to the line of centers, and (b) along it.

4. **REQUIRED** A light inextensible string passes over a light smooth pulley, and carries a mass $4m$ on one end. The other end supports a second light pulley with a string over it carrying masses $3m$ and m on the two ends. Using a suitable pair of generalized coordinates, write down the Lagrangian function for the system, and Lagrange's equations. Find the downward accelerations of the three masses.

ELECTROMAGNETISM

5. Consider a spherical surface of radius R which has a uniform surface charge density $\sigma = \frac{Q}{4\pi R^2}$ everywhere on its surface except for a small cap at the north pole. This cap forms a section of the sphere with polar angle $\theta \leq \alpha$. On the polar cap, the surface charge density is zero. There is no other charge anywhere inside or outside the sphere.

   (a) Write down the two boundary conditions that must be satisfied by the potential on the surface of the sphere.

   (b) Using the result of part (a) show that the potential at an arbitrary point inside the sphere is given by the expression

   $$V_{\text{in}}(r,\theta) = \frac{Q}{8\pi\epsilon_0}\sum_{L=0}^{\infty}\frac{1}{2L+1}\left[P_{L+1}(\cos\alpha) - P_{L-1}(\cos\alpha)\right]\frac{r^L}{R_{L+1}}P_L(\cos\theta). \tag{2}$$

   where r and θ are the radial distance from the center of the sphere and the polar angle measured from the north pole; P_L is the Legendre polynomial of order L.

   (c) Determine the potential at an arbitrary point outside the sphere.

6. A spherical shell of radius R carries a uniform surface charge density σ. The shell is made to spin with angular velocity ω about an axis through its center.

   (a) Determine the surface current density of the rotating shell as a function of position on the shell surface. Choose a coordinate system with the z-axis along the rotation axis and express your result in terms of the polar angle θ. Note that the current density is a vector; your result must specify the magnitude and direction of this vector.

   (b) Find the magnetic moment of the rotating shell.

   (c) Determine the vector potential of the rotating shell at an arbitrary point outside the shell. Express your result in spherical coordinates and give the direction, as well as the magnitude, of the vector potential.

7. **REQUIRED** A parallel plate capacitor is oriented with its plates parallel to the $x - z$ plane. The lower plate at $y = 0$ is positively charged, while the upper plate at $y = d$ is negatively charged. There is a potential difference V between the plates. In addition to the electric field, there is also a uniform magnetic field

in the region between the plates given by $B\hat{z}$. Particles with mass m and charge $q > 0$ are released from rest at $y = 0$ just above the lower, positively charged plate. Assume that the linear extent of the plates is very large compared with d.

(a) Assuming that the motion of the particles is always non-relativistic, write down the equation of motion for the particles under the combined influence of the electric and magnetic fields between the plates. Then show that if $v_x = 0$ at $y = 0$ at time $t = 0$, v_x will subsequently be proportional to y and determine the coefficient of this proportionality.

(b) Noting that magnetic fields cannot change the kinetic energy of the charged particles, write down an equation for the total energy of particles, i.e., kinetic energy plus electric potential energy, as a function of y. Combine this equation with the result of part (a) to derive an equation for v_y as a function of y.

(c) Use the result of part (b) to show that the particles will reach the upper plate only if

$$V \geq \frac{q(Bd)^2}{2m}. \tag{3}$$

THERMODYNAMICS/STATISTICAL PHYSICS

8. An ideal gas starts at a point A in the $P-V$ diagram and undergoes an expansion at constant pressure to reach point B. Pressure is then increased at constant volume to reach a point C. Pressure changes with volume in a linear manner as it starts from C and comes back to A. Let U be the internal energy of the gas, Q be the heat **taken in** by the gas and W be the work **done on** the gas in a certain process.

   (a) Draw the full process $A \rightarrow B \rightarrow C \rightarrow A$ in a $P-V$ diagram.

   (b) What is the sign of dU, Q and W for A to B. Explain your answer.

   (c) What is the sign of dU, Q and W for B to C. Explain your answer.

   (d) What is the sign of dU, Q and W for C to A. Explain your answer.

   (e) What is the sign of dU, Q and W for the full cycle? Explain your answer.

9. Two state paramagnet: Consider a solid made up of a collection of magnetic dipoles. Each dipole can be in one of two states: antiparallel to the applied magnetic field and parallel to the applied magnetic field. Let ϵ and $-\epsilon$ be their respective energies (ϵ can be positive or negative). Let the solid be made up of N dipoles.

   (a) Derive an expression for the number of microstates in the macrostate with $N_\uparrow$ dipoles aligned parallel to the applied magnetic field.

   (b) Write down the expression for the total energy U in terms of N and $N_\uparrow$.

   (c) Assume $N, N_\uparrow, (N - N_\uparrow)$ are all large compared to unity. Prove that

$$U = -N\epsilon \tanh \frac{\epsilon}{kT}. \tag{4}$$

Some useful information

- Stirling's approximation:

$$\ln N! \approx N \ln N - N; \quad \text{for} \quad N >> 1. \tag{5}$$

- Maclaurin series for the exponential function:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \tag{6}$$

- General solution to Laplace's equation in spherical coordinates for a system with azimuthal symmetry is given by

$$V(r,\theta) = \sum_{L=-}^{\infty} \left[A_L r^L + \frac{B_L}{r^{L+1}} \right] P_L(\cos\theta). \tag{7}$$

- Some facts about Legendre polynomials:

$$\int_{-1}^{1} P_L(x) P_{L'}(x) dx = \frac{2}{2L+1} \delta_{LL'} \tag{8}$$

$$(2L+1) P_L(x) = \frac{dP_{L+1}}{dx} - \frac{dP_{L-1}}{dx}; \quad P_{-1} = -1. \tag{9}$$

$$P_L(1) = 1; \quad P_L(-1) = (-1)^L. \tag{10}$$

- The moment of inertia about an axis passing through the center of a solid uniform sphere of radius R and mass M is $\frac{2}{5} M R^2$.

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS

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Ph.D. Qualifying Exam, Fall 2012 – CLASSICAL PHYSICS

FRIDAY, AUGUST 17, 2012 1PM - 5PM

INSTRUCTIONS:

1. There are nine problems on this exam with
   - four in **MECHANICS**;
   - three in **ELECTROMAGNETISM**;
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2. You must solve a total of **SIX** problems with
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MECHANICS

1. **REQUIRED** A wedge shaped block in the shape of an inclined plane with inclination angle α has mass M and rests on a horizontal surface. A smaller block of mass m is placed at the top of the wedge shaped block and released. You can assume all surfaces are perfectly smooth.

   (a) Let V be the magnitude of the wedge's velocity and u be the magnitude of the smaller block's velocity relative to the wedge. Find the magnitude of the smaller block's velocity relative to the Earth.

   (b) Using horizontal momentum conservation and energy conservation, determine the magnitude of

      i. the acceleration of the wedge shaped block and

      ii. the acceleration of the smaller block relative to the wedge shaped block.

   (c) For the case $M = 4m$, calculate the value of α that maximizes the acceleration of the wedge shaped block.

2. A particle of mass m slides without friction on the inside of a cone which has its vertex at the origin, its axis along the z-axis, and whose sides make an angle α with the vertical.

   (a) Choose as independent variables, r, the distance of the particle from the vertex of the cone, and ϕ, the azimuthal angle aroung the axis of the cone, and write down the Lagrangian of the system.

   (b) Derive the Euler-Lagrange equations for the two variables r and ϕ.

   (c) Shown that circular motion inside the cone is possible for any value of r and determine the corresponding angular velocity as a function of r.

3. Consider a horizontal table at some point on the surface of the earth, radius R. Let $\vec{\omega}$ be the angular velocity vector of the earth's rotation about the polar axis. In spherical coordinates the location of the table is given by the angle θ (latitude is $\frac{\pi}{2} - \theta$). Use a local cooridinate system such that the x-axis points South, the y-axis points East, and the z-axis points Up. For a mass m sliding without friction on the table (the local x-y plane), what are the equations of motion for the coordinates $x(t)$ and $y(t)$?

4. A particle, mass m, angular momentum l, moves in a central potential

$$V(r) = \begin{cases} -V_0 & \text{for } r < R \\ 0 & \text{for } r > R \end{cases}, \tag{1}$$

where V_0 and R are positive constants.

(a) Draw a graph of the effective potential.

(b) For which energy range do only bound orbits exist?

(c) For which energy range do both bound orbits and unbound orbits exist?

(d) For which energy range do only unbound orbits exist?

(e) Calculate the polar coordinate representation of the bound orbits.

ELECTROMAGNETISM

5. **REQUIRED** Two circular plates of radius a separated by a distance d form a parallel plate vacuum capacitor that is being charges by a constant current I.

   (a) What is the $\vec{H}$ field between the plates at $\rho = a$?

   (b) At an instant when the charges on the plates are $+q$ and $-q$, what is the electric field between the plates?

   (c) What is the corresponding Poynting vector, $\vec{S}$, between the plates at $\rho = a$?

   (d) Find the surface integral of $\vec{S}$ over the cylindrical surface that forms the boundary of the vacuum region between the plates, and show that this is the same as the rate of change of the stored electrical energy, $\frac{dU}{dt}$, in the capacitor. (Note that $U = \frac{q^2}{2C}$ and $C = \frac{\epsilon_0 \pi a^2}{d}$).

6. Two infinitely long conducting cylinders have their axes parallel and a distance c apart. The radii of the cylinders are a and b where $c >> a$ and $c >> b$. Find the approximate capacitance per unit length of this system.

7. Write down the equation of motion for a charged particle in superimposed uniform, parallel electric and magnetic fields, both in the z-direction, and solve it, given that the particle starts from the origin with velocity $v\hat{x}$. A screen is placed at $x = a$ where $a << \frac{mv}{|q|B}$. Show that the locus of points of arrival of particles with given m and q, but different speeds v, is approximately a parabola. Hoes does this locus depend on m and q?

THERMODYNAMICS/STATISTICAL PHYSICS

8. The dry adiabatic lapse rate:

   (a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for dP/dz, the variation of pressure with altitude, in terms of the density of air, ρ, and acceleration due to gravity, g.

   (b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. Show, then, that the pressure obeys the **barometric equation**

   $$\frac{dP}{dz} = -\frac{mg}{kT}P. \tag{2}$$

   (c) Prove that

   $$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}. \tag{3}$$

   for an ideal gas with f degrees of freedom expanding adiabatically. You can assume the equipartition theorem.

   (d) Solve for $T(z)$ using (2) and (3).

9. Assume that a solid can be modelled as a collection of identical oscillators with quantized energy. Let ϵ be the unit of energy in the Einstein solid.

   (a) Derive an expression for the number of microstates, $\Omega(N,q)$, in the macrostate with total energy $q\epsilon$ for an Einstein solid with N particles.

   (b) Assume N and q are both large to show that

   $$\Omega(N,q) \approx \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N. \tag{4}$$

   (c) Prove that the total energy is given by

   $$U = q\epsilon = \frac{N\epsilon}{e^{\frac{\epsilon}{kT}} - 1}. \tag{5}$$

   (d) Derive an expression for the heat capacity, $C = \frac{dU}{dT}$, and show that

   $$\lim_{T\to\infty} C = Nk. \tag{6}$$

Some useful information

- Stirling's approximation:

$$\ln N! \approx N \ln N - N; \quad \text{for} \quad N >> 1. \tag{7}$$

- Maclaurin series for the exponential function:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \tag{8}$$

- $$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x. \tag{9}$$

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS

UNIVERSITY PARK, MIAMI, FL 33199

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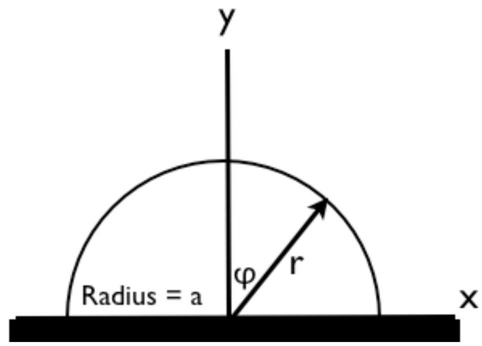
1. Mechanics

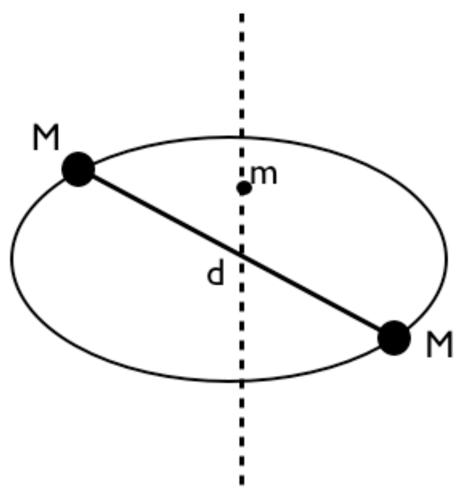


Consider a mass that starts at the very top of a hemisphere of radius *a* as shown in the above figure. The mass slides down the hemisphere without friction. Use of the method of Lagrange multipliers is advised to solve this problem.

(a) Write down the Lagrangian in terms of the two generalized coordinates *r* and ϕ. Include the constraint in terms of the Lagrange multiplier λ.

(b) Solve the problem to find the angle ϕ at which the mass falls off.

2. Mechanics



Two stars, each of mass M and separated by a distance *d*, orbit about their center of mass. A planetoid of mass *m,* (*m* << M) moves along the axis of this system perpendicular to the orbital plane. Let T_p be the period of oscillations for the planetoid for small displacements about the center of mass of the two star system. Let T_s be the period of motion of the two stars about the center of mass of the two star system. Determine the ratio T_p/T_s.

3. Mechanics **REQUIRED**

An object is released from rest at a distance r_0 (larger than the radius of the Earth) from the Earth's center.

(a) Using energy conservation, show that the radial velocity component of the object when it has fallen to the height r is given by (negative because it is moving toward the Earth's center) $v = - [\ 2GM(r_0 - r)/(r\ r_0)\ ]^{1/2}$ where M is the mass of the Earth.

(b) Use the result of part (a) to show that if r_0 is so large that the Earth's radius can be neglected, the time required for the object to reach the Earth's surface (that is $r \cong 0$) is given by $t = \pi\, T$ where $T = (1/2\ r_0)^{3/2} /(GM)^{1/2}$. You may need the integral $\int x^2/(a^2-x^2)^{1/2}\, dx = 1/2\ [\ -x\ (a^2 - x^2)^{1/2} + a^2 \sin^{-1}(x/a)\ ]$.

(c) Find the time required for the object to fall to $r = 1/2\ r_0$.

4. Mechanics

A useful potential energy function to describe the interaction between two atoms separated by a radial distance is the Morse potential given by

$$U(r) = D\ [\ 1 - e^{(-a\{r - r_0\})}\ ]^2$$

where D is positive and has units of energy, r_0 is a positive distance, a is positive with units of inverse length and $(ar_0) >> 1$.

(a) Find all equilibrium points of this potential.

(b) Determine which of the equilibrium points are stable.

(c) Sketch the potential labeling all the equilibrium points.

(d) Calculate the energy required to fully separate two atoms ($r \rightarrow \infty$) that were initially in equilibrium at one of the stable points.

(e) Obtain the frequency of small oscillations about all stable equilibrium points for two identical atoms of mass m bound to each other by this potential.

5. Electromagnetism

A very long solenoid with radius a and its axis along the z-axis has n turns per unit length and carries a current I. Coaxial with the solenoid near its middle is a circular loop of wire with radius b, $b >> a$, and electrical resistance R. The current in the solenoid is gradually decreasing at a constant rate dI/dt (negative).

(a) Using Faraday's law, find the current induced, I_{ind}, in the circular loop in terms of dI/dt. What is the direction of the induced current?

(b) Determine the electric field induced at the surface of the solenoid due to the changing magnetic flux through the interior of the solenoid. What is the direction of **E**?

(c) Calculate the Poynting vector just outside the solenoid surface that is associated with the electric field determined in part (b) and the magnetic field arising from the induced current in the loop. Assume that the radius b is so large compared to the radius a that the magnetic field from the loop just outside the solenoid is well approximated by the field on the loop axis.

(d) Integrate the result of part (c) over the surface of the solenoid and thereby show that the total power flowing out of the solenoid is just equal to I_{ind}^2R. Note that the integral over the solenoid length should be taken from $-\infty$ to ∞. You may need the integral $\int dz /(b^2 + z^2)^{3/2} = z/[b^2 (b^2 + z^2)^{1/2}]$.

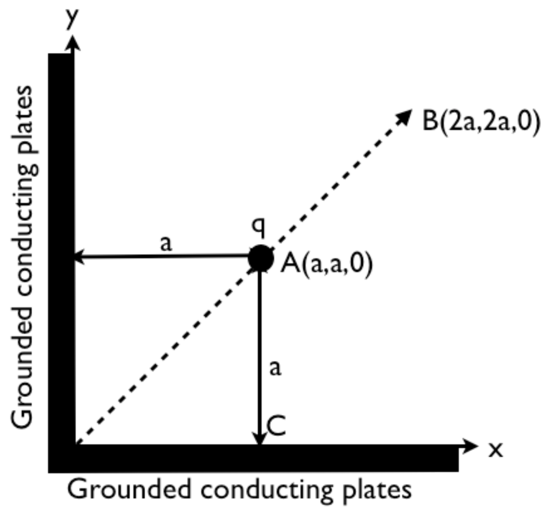
6. Electromagnetism

The potential at the surface of a hollow sphere with radius R is given by

$$V(R,\theta) = k \sin^2 (\theta/2)$$

where k is a constant. Using the solutions to Laplace's equation and the appropriate boundary conditions, determine the electric potential at all points inside and outside of this spherical hollow shell.

7. Electromagnetism **REQUIRED**



A charge *q* is placed in the region between two grounded conducting planes meeting at a right angle. The planes are very large and could be considered as extending to infinity for this problem. Initially the charge is located at the position A(a,a,0).

(a) Sketch the electric field lines everywhere.

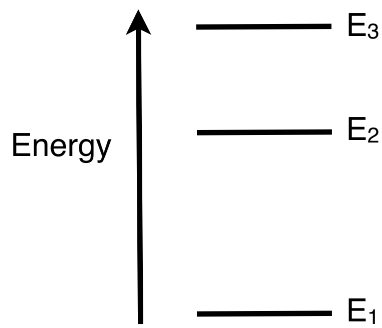
(b) Calculate the net force on *q* when it is at point A(a,a,0).

(c) When the charge is at A(a,a,0), calculate the induced surface charge density at C(a,0,0).

(d) How much work do you need to do in order to move the charge from A(a,a,0) to B(2a,2a,0) along the diagonal line?

8. Thermodynamics/Statistical Physics

A solution in equilibrium at absolute temperature T contains a large number of the same type of protein molecules. The internal states of a protein molecule are arranged as follows:



Using the fundamental definition of entropy, $S = -R \sum_j P_j \, ln \, P_j$, determine an expression for the Helmholtz free energy of the internal states of one mole of these protein molecules. Your expression should be expressed in terms of E_1, E_2, E_3 and physical constants. Collect terms to simplify your answer.

9. Thermodynamics/Statistical Physics

The differential form of the First Law is $du = Tds - Pdv$, where u, T, s, P and v are internal energy, temperature, entropy, pressure and volume respectively.

(a) Define Gibbs energy $G = u - Ts + Pv$ and write the differential form of the First law for Gibbs free energy.

(b) Under 'standard' conditions, (i.e. $T=298K$, $P = P_0 = 1.013 \times 10^5\ N/m^2$) water is in liquid phase. But one can cause water to boil (i.e. phase transformation to vapor state) by reducing the pressure. Use the differential form obtained in part (a) to calculate the evaporation pressure P' at $T=298K$. (Treat water vapor as an ideal gas and water as an incompressible liquid, and assume $P'/P_0 << 1.0$). [Molar data: G(liquid) = $-285.8\ kJ$, G(vapor) = $-241.8\ kJ$, v(liquid) = $10\ cm^3$, and $R = 8.31\ J/(mol\ K)$].

Classical Physics
Ph.D. Qualifying Exam   Fall 2014
Florida International University Department of Physics

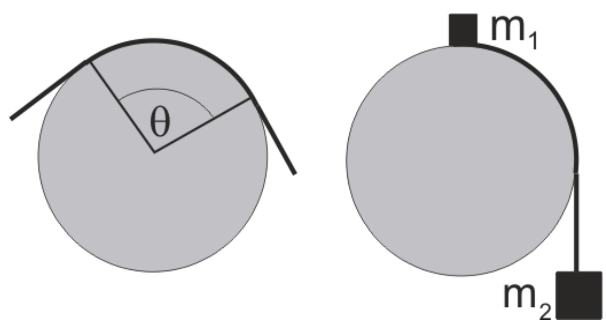
Instructions: There are nine problems on this exam. Three on mechanics (Section A), four on electricity and magnetism (Section B), and two on thermodynamics and statistical physics (Section C). You must solve a total of six problems with at least two from Section A, two from Section B, and one from Section C. You must also do the problems marked **Required**.

Do each problem on its own sheet (or sheets) of paper. Turn in only those problems you want graded. Write your student ID number on each page but not your name.

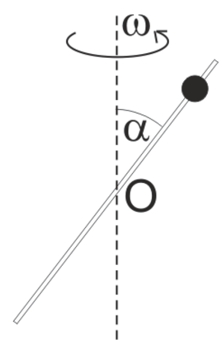
You may use a calculator and the math handbook as needed.

Section A

1. **Required**
   a) A massless string is in contact with a section of a rough cylindrical surface as shown on the left. Show that if a force F is applied at one end of the string, the maximum force that can be applied at the other end is $Fe^{\mu\theta}$ if the string is to remain at rest (μ is the coefficient of static friction between the rope and surface).
   b) The diagram on the right shows two masses connected by a string, one hanging, the other resting at the highest point of a rough cylindrical surface. If the coefficient of static friction for the resting mass is also μ, and the system is just about to slide, what is the ratio ${}^{m_2}/_{m_1}$?



2. A smooth rod rotates with constant angular velocity ω about a vertical axis and is inclined at an angle α to the axis. A bead of mass m slides freely and without friction on the rod, and gravity acts vertically downwards.
   a) Obtain the equation of motion of the bead using Lagrangian methods.
   b) Find the distance along the rod from O at which the bead can remain at rest relative to the rod.

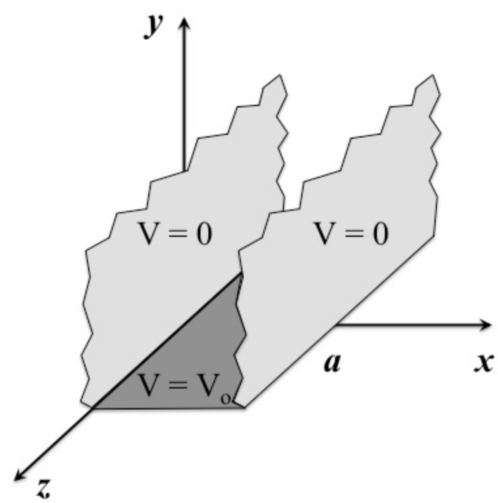


3. Provide at least two corrections to the reading on your bathroom scale in Miami to estimate your true mass. Describe the mathematical details. Estimate the size of each of your corrections using readily available data.

Earth's equatorial radius = 6,378 km
Earth's polar radius = 6,357 km
Atmospheric pressure = 1.01×10^5 Pa
Density of air= 1.2754 kg/m^3
Miami latitude = 25.8° North
g = 9.81m/s^2

Section B

4. **Required** Two infinitely large metal plates lie parallel to the y-z plane, one at $x = 0$ and one at $x = a$, as shown in the figure to the right. These two plates are maintained at zero potential ($V = 0$) and both extend from $y = 0$ to $y = +\infty$ and from $z = -\infty$ to $z = +\infty$. A third plate, this one maintained at a constant potential V_0, lies in the x-z plane and forms the bottom of a "slot". Determine an expression for the potential $V(x,y)$ for any point within the "slot". Notice that due to symmetry, the potential is independent of z.



5.

a) Using Maxwell's equations, show that the electromagnetic wave equations in charge free and current free space are given by

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

and

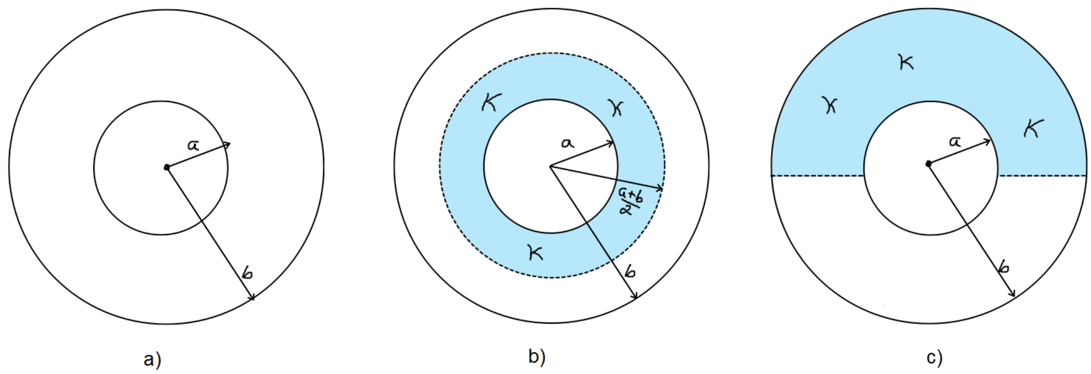
$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

.

b) Using the results of part a), determine the speed of EM waves. What is the physical significance of this?

c) Write down the real electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω and phase angle zero that is travelling in the positive x direction and polarized in the z direction. Show that these fields are solutions to the wave equation.

d) Using the electric and magnetic fields from part c), determine the Poynting vector. Explain the physical significance of the Poynting vector.



6. Consider the configurations of three concentric **spherical** capacitors in the figure above. For maximum credit please explain your reasoning.

a) In figure a) the concentric conducting spheres have an inner radii a, outer radii b and the region between the spheres is vacuum. Find the capacitance of the configuration.

b) In figure b) a dielectric material with a dielectric constant κ fills up the volume between the spheres up to a radius of $(a+b)/2$. Find the total capacitance of this configuration

c) In figure c) the dielectric is all located on the upper hemisphere in the region between the spheres as shown. Find the total capacitance of this configuration.

7. Consider an (infinitely) long line of charge at rest with respect to frame O' lies along the x-axis. In frame O', the charge per unit length is given by λ'.

a) Finding the electric field in the frame O' is an electrostatics problem. Use Gauss's law ($\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$) to find the components of the electric field in the frame O'. Use the cylindrical coordinates (x, ρ, φ).

b) Calculate the components of the magnetic field, B_x, B_ρ, B_φ in frame O'.

Let O be another inertial frame, moving with respect to frame O' at constant velocity $v\hat{\imath}$.

c) Calculate the linear charge density, λ, in the frame O in terms of λ'.

d) Calculate the components of the electric field, E_x, E_ρ, E_φ in frame O.

e) Calculate the components of the magnetic field, B_x, B_ρ, B_φ, in frame O using the Lorentz transformation.

f) Use Ampere's law ($\int \vec{B} \cdot d\vec{l} = \mu_0 I$) to calculate $\vec{B}$, and compare to your answer for part d). Note that the current is $I = \frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \lambda v$.

Section C

8. The complete thermodynamic cycle of a heat engine using an ideal gas with constant specific heat capacities consists of four steps. Step A is an *adiabatic* compression from pressure P_1 and volume V_1 to pressure P_2 and volume V_2. Step B is an *isobaric* expansion at pressure P_2 from volume V_2 to volume V_3. Step C is an *adiabatic* expansion from pressure P_2 and volume V_3 to pressure P_1 and volume V_4. Step D is an *isobaric* compression at pressure P_1 from volume V_4 back to the original volume V_1.

a) Make a PV diagram for the complete cycle.

b) Show that the ratio of the heat flow out of the engine during step D to the heat flow into the engine during step B is given by
$Q_{out}/Q_{in} = (T_4\text{-}T_1)/(T_3\text{-}T_2)$

c) The efficiency of the engine is defined as the ratio of the work done by the engine to the input heat transfer. Use the ideal gas equation together with the fact that PV^{γ} is constant for an adiabatic process, where γ is the adiabatic index of the gas, to show that the efficiency of the engine is given by
$e = 1 - \left(\frac{P_1}{P_2}\right)^{\alpha}$,
where $\alpha = \frac{(\gamma-1)}{\gamma}$.

9. Consider a system with two non-degenerate energy levels with energies ε_0 and ε_1, where $\varepsilon_1 > \varepsilon_0 > 0$. Suppose that the system contains N distinguishable particles at temperature T, so that the system is described by classical Boltzmann statistics.

a) Show that the average energy per particle in the system is given by the expression
$\langle u \rangle = {}^{U}\!/_{N} = (\varepsilon_0 + \varepsilon_1 e^{-\alpha})/(1 + e^{-\alpha})$,
where $\alpha = (\varepsilon_0 - \varepsilon_1)/k_B T$ with k_B denoting Boltzmann's constant.

b) show that the constant volume heat capacity per particle is given by
$\frac{C_V}{N} = k_B \alpha^2 e^{-\alpha}/(1 + e^{-\alpha})^2$.

c) Show that C_V/N goes to zero as T goes to zero, and that as the temperature becomes very large, $C_V/N \cong \frac{1}{4} k_B \alpha^2$.

Classical Physics
Ph.D. Qualifying Exam Fall 2015
Florida International University Department of Physics

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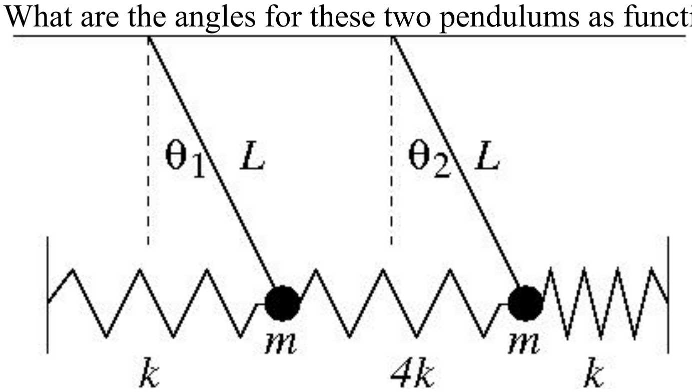
You may use a calculator and the math handbook as needed.

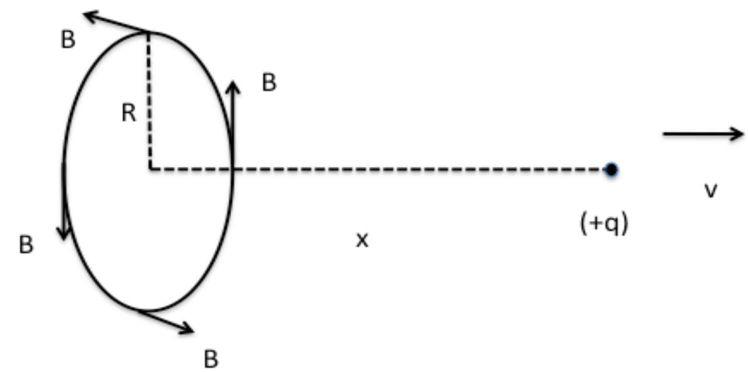
Section A

1. **Required** A block of mass m is placed inside a box of mass M, which is then hung from a spring with spring constant k. The system is pulled down some distance d and released at time $t=0$. Determine the reaction force between the block and the bottom of the box as a function of time. For what value of d will the block just lose contact with the bottom of the box when at the top of the vertical oscillation?
2. A particle of mass m rests on a smooth plane that is initially horizontal. The plane is raised to an inclination angle θ at a constant rate α (so $\theta = 0$ at $t = 0$), causing the particle to move down the plane. Determine the motion of the particle. Hint: It is best to place your origin at the fixed end of the plane and define the distance r from the origin as one generalized coordinate and the angle of inclination θ as the other. Your differential equation for the motion will have a general solution to the homogeneous part (r_h) and a particular solution to the non-homogeneous part (r_p).
3. Two identical pendulums (length L) are connected and confined by three massless springs with the identical neutral length but different spring constants (k for the left and the right springs, $4k$ for the middle spring). At equilibrium positions (bottom), the springs are not compressed or stretched. Originally, the left pendulum is at $-3\theta_0$, and the right pendulum is at $2\theta_0$, then both are released from rest. (Assume small angle oscillations). What are the angles for these two pendulums as functions of time?



Section B

4. A moving charged particle (+q) produces a magnetic field, according to Biot Savart law. On the other hand, Ampere's law says the change of electric field can also induce a magnetic field. Demonstrate that the magnitude of the magnetic field (as a function of v, x, q and R) at the circle shown below, obtained from the two laws, are consistent with each other.

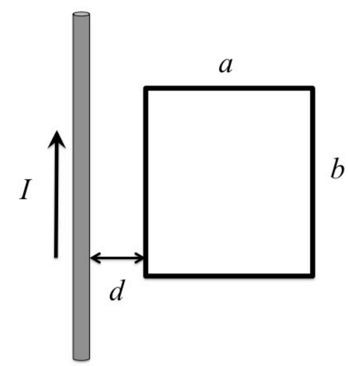


5. The potential at the surface of a hollow sphere with radius R is given by

$$V_o(\theta) = k\cos(3\theta)$$

where k is a constant and θ is the polar angle. Using the solutions to Laplace's equation and the appropriate boundary conditions, determine the electric potential inside and outside of this spherical hollow shell and the surface charge density σ(θ).

6. A rectangular loop with dimensions a x b is placed a distance d from a long, straight wire carrying current I.



(a) Determine the total flux through the rectangular loop.
(b) If the current in the long wire is increased steadily at a rate of $dI/dt = k>0$, determine the induced emf in the loop.
(c) If the loop has total resistance of R, determine the magnitude and direction of the induced current.

7. (**Required**) A rectangular wave-guide with height $\Delta x = a$ and width $\Delta y = b$ runs along the z-axis as shown in the figure.

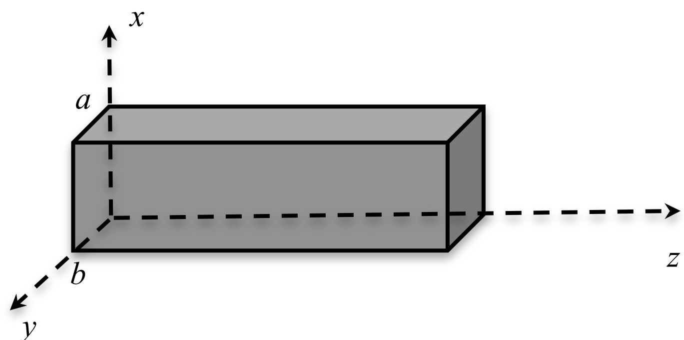
(a) Show that the longitudinal magnetic fields (TE modes) are given by

$$B_z = B_0 \cos(m\pi x/a)\cos(n\pi y/b)$$

where $m = 0, 1, 2, \ldots$ and $n = 0, 1, 2\ldots$

(b) Show that the cutoff frequency is given by

$$\omega_{mn} = \sqrt{(m/a)^2 + (n/b)^2}.$$



(c) Suppose the wave guide has dimensions 3.0 cm x 1.0 cm. What TE modes will propagate in this wave guide if the driving frequency is 2×10^{10} Hz?

Section C

8. The van der Waals equation of state describes an interacting gas that can undergo a phase transition to the liquid state. It has the form

$$(P + aN^2/V^2)(V - bN) = Nk_BT$$

where P, V, and T are the pressure, volume, and temperature of the gas, N is the number of molecules in the gas, k_B is Boltzman's constant, and a and b are constants. At high temperatures, the pressure always increases with decreasing volume, and no phase transition is possible. At low temperatures, there is an unstable region where the pressure and volume both decrease at the same time, and a phase transition can occur to the liquid phase. These two regions are separated by a critical point characterized by critical values P_C, V_C, and T_C of the pressure, volume, and temperature of the gas.

a) At the critical point, both the first and second partial derivatives of the pressure with respect to the volume vanish. Use these conditions to derive expressions for P_C, V_C, and T_C in terms of a, b, k_B, and N.

b) Define dimensionless pressures, volumes, and temperatures through the relations
$v=V/V_C$, $p=P/P_C$, and $t=T/T_C$. Show that in terms of these dimensionless quantities, the van der Waals equation of state reads

$$(p+3/v^2)(v-1/3) = (8/3)t$$

9. A system has a large number of single particle states M. Particles can be added to the system, but the particles are fermions so that any single particle state can be

occupied by either zero or one particle. To add a particle into a state in this system requires an energy ε. All M single particle states are degenerate with the same energy ε:

ε ____ ____ ____ ____ ____ ____ ____ . . . ____

1 2 3 4 5 6 7 M

The system is in equilibrium at temperature T so that each single particle state is equally likely to be occupied. The temperature is high enough so that particle number N>>1, but M>>N. Determine an analytical expression for how the natural logarithm of the partition function varies as a function of N that depends on $\beta=1/kT$, N, M, and ε; i.e. calculate $\alpha=d(\ln Z)/dN$.