Modern Physics **Name** ____________________

Ph.D. Qualifying Exam, Spring 2006

INSTRUCTIONS: There are nine problems on this exam: Four in section A, and five in section B. You must solve SIX problem with at least two from each section. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: A highly energetic photon (wavelength λ) collides with an electron at rest. After the collision the angle θ between the incident photon direction and the final direction is measured. Calculate the change in the wavelength, $\Delta\lambda = \lambda' - \lambda$ in terms of θ, where λ' is the scattered photon wavelength. What is particularly remarkable about this result?

Problem A2: Consider the relativistic wave operator restricted to one spatial dimension and time:

$$\nabla^2 = \frac{\partial^2}{\partial\,(ct)^2} - \frac{\partial^2}{\partial x^2}.$$

Find the most general coordinate transformation of the form

$$x' = a_{11} + a_{14}ct \qquad ct' = a_{41}x + a_{44}ct$$

that leaves ∇^2 invariant.

Problem A3: Two relativistic particles of mass m_1 and m_2 collide head on with speeds v_1 and v_2 as seen by a stationary observer. After colliding they form a single particle of mass M moving with speed V relative to the observer. Find M and V in terms of m_1, m_2, v_1, and v_2. Is it possible for the formed particle to be a photon if neither m_1 or m_2 are zero?

Problem A4: Consider a particle of mass m oscillating harmonically between $-a \leq x \leq a$. Use the Wilson-Sommerfeld quantization rule to show that $E_n = nh\nu$ where E_n is the energy of the nth state, h is Planck's constant, and ν is the frequency of the oscillator.

Problem B1: Consider the motion of a free non-relativistic particle of mass m in one dimension. Let its wave function at $t = 0$ be

$$\Psi(x,0) = \left[\frac{2}{\pi\sigma^2}\right]^{\frac{1}{4}} e^{-\frac{x^2}{\sigma^2}}.$$

Obtain an expression for the particle's wave function $\Psi(x,t)$ for all $t > 0$. Show that for all t, $\langle x \rangle = 0$ and that the uncertainty in x is given by

$$\Delta x(t) = \sqrt{\frac{\sigma^2}{4} + \left[\frac{\hbar t}{m\sigma}\right]^2}.$$

Problem B2: Two spin-$\frac{1}{2}$ particles form a composite system. Particle 1 is in an eigenstate of $S_{1z} = +1/2$ and particle 2 is in an eigenstate of $S_{2x} = +1/2$. What is the probability that a measurement of the total spin will give a value of zero? You may wish to utilize the Pauli spin matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Problem B3: The interaction Hamiltonian H among three states $|1\rangle$, $|2\rangle$, and $|3\rangle$ is given by

$$\hat{H} = \begin{pmatrix} 0 & a & a \\ a & 0 & a \\ a & a & 0 \end{pmatrix}$$

Find the eigenvalues and the corresponding orthogonal eigenfunctions of H.

Problem B4: Consider an infinite square well potential with a slight taper at the bottom:

$$\begin{array}{ll} V = \frac{\epsilon}{a}|x| & \text{for } |x| < a \\ V = \infty & \text{for } |x| > a. \end{array}$$

Calculate the ground state energy of this particle to first order in ϵ.

Problem B5: A particle of mass m is incident from the left on a step potential:

$$V(x) = \begin{array}{ll} 0, & x < 0 \\ V_0, & x \geq 0 \end{array}$$

where $V_0 > 0$. If the particle is found to be reflected with a probability of $1/9$, find the incident kinetic energy in terms of V_0.

Modern Physics **Name** ______________________

Ph.D. Qualifying Exam, Fall 2006

INSTRUCTIONS: There are nine problems on this exam: Four in section A, and five in section B. You must solve SIX problems doing at least two from each section. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: A particle with rest mass m_0 has a velocity $(v_x, 0, 0)$ in the (x, y, z, t) coordinate frame. Write down the expressions for the particle's energy and momentum in this frame. Let the coordinate frame, (x', y', z', t'), be related to (x, y, z, t) by a Lorentz boost of $\beta = v/c$ along the x-direction. Find the particle's energy and momentum in the (x', y', z', t') frame and express these two quantities solely in terms of v, c and the corresponding quantities in the (x, y, z, t) frame.

Problem A2: A pion at rest decays into a muon plus a neutrino. Find the rest-frame speed of the muon in terms of the muon and pion masses (m_μ and m_π) and the speed of light c. You can assume that the neutrino is massless.

Problem A3: Prove the classical relation for the average energy of blackbody radiation: $\bar{E} = kT$. Use a Maxwell-Boltzmann energy distribution function: $f(E) = Ce^{-E/kT}$.

Problem A4: In this problem you are going to determine the phase and group velocity, v_{ph} and v_g, respectively, of a relativistic particle. First, you need to show that

$$v_{ph} = \frac{\omega(k)}{k} = \frac{E(p)}{p} \quad \text{and} \quad v_g = \frac{d\omega(k)}{dk} = \frac{dE(p)}{dp}$$

The first equality in both of these equations can be assumed. With these relationships, determine v_{ph} and v_g. Write these in terms of the ordinary particle velocity v. Recall that the energy of a relativistic particle is related to its momentum by

$$E^2 = p^2c^2 + m^2c^4$$

and that the relativistic momentum is given by

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

Problem B1: Consider a non-relativistic particle of mass m under the influence of the potential,

$$V(x) = -\frac{\hbar^2}{m}[\delta(x) + \delta(x+d)].$$

Show that there are two bound states if $d > 1$ and that there is only one bound state if $d < 1$.

Problem B2: Prove Ehrenfest's theorem in one dimension:

$$\frac{d\langle p\rangle}{dt} = \langle -\frac{dV}{dx}\rangle.$$

Recall that $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$.

Problem B3: A particle in an infinite square well of width a has the initial wave function

$$\Psi(x,0) = Ax(a-x).$$

a) Normalize $\Psi(x,0)$.

b) Find $\Psi(x,t)$. Recall that completeness tells us

$$\Psi(x,0) = \sum_n c_n f_n(x)$$

and

$$c_n = \langle f_n|\Psi\rangle.$$

Problem B4: Starting with the canonical commutation relations for position and momentum ($[x,p_x]$, $[y,p_y]$, $[z,p_z]$, etc...), work out the commutators

$$[L_x, j] \quad [L_x, p_j],$$

for $j = x, y, z$. Use these results to obtain

$$[L_x, L_j],$$

where you will recall $L_x = yp_z - zp_y$, $L_y = zp_x - xp_z$, and $L_z = xp_y - yp_z$.

Problem B5: Two identical fermions are placed in an infinite square well. They interact weakly with each other through the potential

$$V(x_1, x_2) = -aV_0\delta(x_1 - x_2),$$

where V_0 is a constant with dimensions of energy and a is the width of the well.

a) Ignoring the interaction between the particles, find the ground state and wave functions and energy.

b) Use first-order perturbation theory to estimate the effect of the particle-particle interaction on the energy of the ground state. Explain how you could have gotten this answer without using perturbation theory.

Modern Physics **Name** ______________________

Ph.D. Qualifying Exam, Fall 2007

INSTRUCTIONS: There are nine problems on this exam: Four in section A, and five in section B. You must solve SIX problem with at least two from each section. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: In a two-body scattering event, $A+B \to C+D$, it is convenient to introduce Mandelstam variables

$$s = \frac{(p_A+p_B)^2}{c^2}, \quad t = \frac{(p_A-p_C)^2}{c^2}, \quad u = \frac{(p_A-p_D)^2}{c^2},$$

where p_A, p_B, p_C and p_D are the four momenta of A, B, C, and D respectively. The squares in the numerators are the relativistically invariant dot product. Let m_A, m_B, m_C and m_D denote the rest masses of A, B, C and D respectively.

a) Show that $s+t+u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

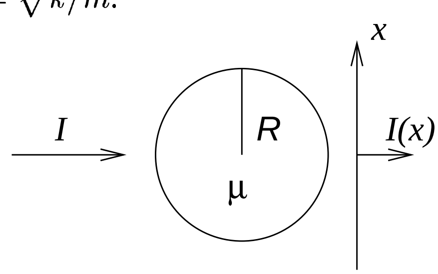
b) Show that the center of mass energy of A is $\frac{(s+m_A^2-m_B^2)c^2}{2\sqrt{s}}$.

c) Show that the lab energy of A (B at rest) is $\frac{(s-m_A^2-m_B^2)c^2}{2m_B}$.

d) Show that the total Center of Mass energy is $\sqrt{s}c^2$.

Problem A2: Use the Bohr quantization rule for angular momentum ($L = mvr = n\hbar$) to calculate the energy levels of a harmonic oscillator. Consider circular orbits only. Harmonic oscillators have a central force given by Hooke's law: $F = -kr$. Write your answer in terms of the angular frequency $\omega = \sqrt{k/m}$.

Problem A3: A solid cylinder with radius R, density ρ and absorption coefficient μ (in units of area over mass) is being imaged by X-rays. The incident intensity is I_0. Derive a formula for the intensity $I(x)$ in the image plane.



Problem A4: Show that the speed of an electron in terms of β can be expressed as

$$\beta = \sin \arccos\left(\frac{1}{K/m_e + 1}\right),$$

where K is the kinetic energy and m_e is the electron mass.

Problem B1: A particle of mass m is in the state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]},$$

where A and a are positive real constants.

a) Normalize the wave function

b) For what potential energy function $V(x)$ does Ψ satisfy the Schrodinger equation?

c) Find the uncertainties in the position and momentum, σ_x and σ_p, and see if their product is consistent with the uncertainty principle.

Problem B2: Consider a bead of mass m that slides on a frictionless wire ring of circumference a. This is almost like a free particle only that $\psi(x) = \psi(x+a)$. Find the stationary states and the corresponding energies. Discuss any degeneracies.

Problem B3: Consider a 3-D vector space spanned by an orthonormal basis $|1\rangle$, $|2\rangle$, and $|3\rangle$. Ket $|\alpha\rangle$ and $|\beta\rangle$ are given by

$$|\alpha\rangle = i|1\rangle - i|2\rangle - 2|3\rangle$$

and

$$|\beta\rangle = i|1\rangle + 2|2\rangle.$$

a) Construct $\langle\alpha|$ and $\langle\beta|$ in terms of the duel basis $\langle 1|$, $\langle 2|$, and $\langle 3|$.

b) Find $\langle\alpha|\beta\rangle$, and $\langle\beta|\alpha\rangle$, and confirm $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle$

Problem B4: An electron is at rest in a uniform magnetic field $\vec{B} = B_x\hat{\imath} + B_z\hat{k}$ $(B_x >> B_z)$. The Hamiltonian is

$$H = H_0 + H' = \frac{eB_x}{m}S_x + \frac{eB_z}{m}S_z.$$

Find the first-order perturbation corrections of the electron wave functions.

Problem B5: Consider the one dimensional potential

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } 0 < x < a \\ 0 & \text{for } x > a, \end{cases}$$

with $V_0 > 0$. Assuming an incident stream of particles from the far left $(x \to -\infty)$ with a monochromatic energy of $E = \hbar\omega$ and a current of $I = 1$, obtain an expression for the reflected current R and the transmitted current T. Also obtain an expression for the current in the region $0 < x < a$. Work these out for the two cases, $E > V_0$ and $E < V_0$. Discuss the physical interpretation of your results.

Modern Physics **Name** ____________________

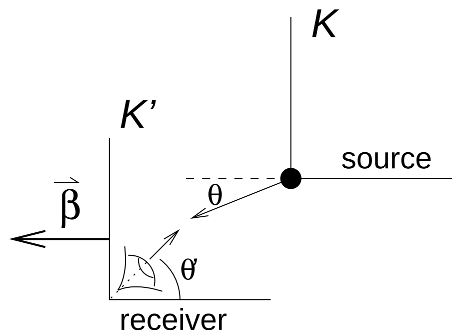
Ph.D. Qualifying Exam, Fall 2008

INSTRUCTIONS: There are nine problems on this exam: Four in section A, and five in section B. You must solve SIX problem with at least two from each section. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: The energy of a photon is given by $E = h\nu$ and its momentum is given by $p = h\nu/c$ (or $p = E$ setting $c = 1$), where ν is the frequency of the emitted light.

a) Use the Lorentz transformation of the energy to find the frequency, ν', of the light observed by a receiver moving *directly toward* the source. What is the frequency when the receiver is moving *directly away* from the source.

b) Now consider the case where the light is emitted at an angle of θ in the frame of the source and observed at an angle of θ' at the receiver.



a) Find the Doppler shifted frequency.

b) Show that

$$\tan\theta' = \frac{\sin\theta}{\gamma\,(\cos\theta - \beta)}.$$

Problem A2: A photon of energy E scatters off of a free electron (mass m_e) through an angle of θ. Suppressing factors of c,

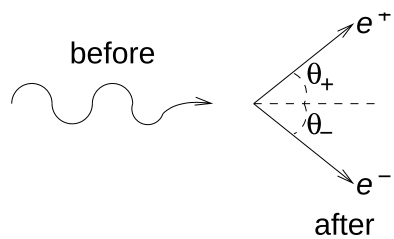
a) show that the energy of the scattered photon is given by

$$E' = \frac{E m_e}{m_e + E(1 - \cos\theta)}.$$

b) Show that the kinetic energy of the electron after the collision is given by

$$T_e = \frac{E^2(1-\cos\theta)}{m_e + E(1-\cos\theta)}.$$

Problem A3: Show that an isolated photon cannot be converted into an electron/positron pair: $\gamma \to e^- + e^+$. Do not assume the angles of the electron and positron are equal.



Problem A4: In the standard derivation of the Bohr formula, the nucleus is stationary and, therefore, the implicit assumption is that the mass of the nucleus is infinite. However, the electron actually orbits around the center of mass of the electron-nucleus system. The proper derivation would replace the electron mass with the *reduced mass* of the system given by

$$\mu = \frac{m_e M}{m_e + M},$$

where M is the nuclear mass. Use μ to find the correct expression for the Bohr energy E_n. Show that the relative error in the ground-state energy of hydrogen is given by

$$\frac{\Delta E}{E} = \frac{m_e}{m_p},$$

where ΔE is the difference between the energy assuming a stationary nucleus and the energy for finite-mass nucleus.

Problem B1: Prove that for normalizable solutions to the Schrodinger equation, the separation constant E must be real. Hint: Write E as $E_0 + i\Gamma$ (with E_0 and Γ real), and show that if the normalization condition is to hold for all t, Γ must be zero.

Problem B2: A particle of mass m approaches an abrupt potential drop:

$$V(x) = \begin{cases} 0, & \text{for } x < 0, \\ -V_0, & \text{for } 0 \le x. \end{cases}$$

What is the reflection coefficient if $E = V_0/3$

Problem B3: A particle is in a cubical well:

$$V(x) = \begin{cases} 0, & \text{for } x, y, z \text{ between 0 and } a, \\ \infty, & \text{elsewhere.} \end{cases}$$

Find the stationary states and the corresponding energies.

Problem B4: Find all the energy levels of a quantum particle of mass m moving under the effect of a two dimensional potential of the form $\frac{1}{2}m\omega^2(x^2 + 2y^2 + \sqrt{3}xy)$. Are there any degeneracies?

Problem B5: Consider a particle of mass m in a three-dimensional central potential of the form

$$V(r) = \begin{cases} 0, & \text{if } a \le r \le b, \\ \infty, & \text{elsewhere.} \end{cases}$$

This corresponds to a particle trapped between two hard spherical shells. Find the $l = 0$ total wave function and energy.

Modern Physics **Name** ______________________________

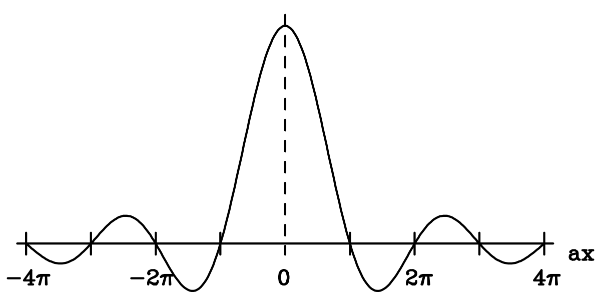
Ph.D. Qualifying Exam, Fall 2009

INSTRUCTIONS: There are nine problems on this exam: Five in section A, and four in section B. You must solve a total of SIX problems with at least two from each section. You must also solve the two problems marked with *. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: Show that the speed of the electron in the nth Bohr orbit of a hydrogen atom is given by $v = c\frac{\alpha}{n}$ where $\alpha = ke^2/\hbar c$ is the fine structure constant.

Problem A2: Consider a wave packet that is formed by the momentum distribution:

$$g(k) = \begin{cases} 0 & k < -a \\ C & -a \le k \le a \\ 0 & a < k \end{cases}$$



The resulting wave function, $f(x)$ looks like the figure to the right.

a) Find the coordinate-space wave function $f(x)$.

b) Find C such that the wave function is normalized.

c) Show that a reasonable definition of Δx for your answer to $f(x)$ gives $\Delta k \Delta x > 1$ independent of C.

Problem A3*: An electron with unknown energy scatters elastically from a proton target. The angles and momenta of the scattered particles are all measured. Show that the incident energy, E_e, can be determined two ways:

a) $E_e = p_{e'} \cot\theta_e + p_p \cot\theta_p$

and

b) $E_e = m_p \left(\cot\frac{\theta_e}{2}\cot\theta_p - 1\right)$,

where $p_{e'}$ and p_p are the momenta of the scattered electron and proton, respectively and θ_e and θ_p are their respective scattering angles. Assume that the the incident energy is much greater than the electron mass ($E_e \gg m_e$). You may also need

$$\cot\frac{A}{2} = \frac{\sin A}{1 - \cos A}.$$

Problem A4: A photon torpedo is fired horizontally from the starship FIUSS Madique at a (very long) rocket ship that is moving vertically with a speed of $\frac{4}{5}c$. One second later, the Madique fires a second torpedo at the rocket ship.

a) In the reference frame of the Madique, how far apart are the impact points of the two torpedoes?

b) In the reference frame of the rocket ship, how far apart are the impact points?

Problem B1: A non-relativistic particle is in a one-dimensional infinite square well. Show that the frequency of radiation emitted for the transition n to $n-1$ equals the frequency of motion when $n \gg 1$. What principle does this satisfy and why is it important?

Problem B2: Two particles of mass m are attached to the ends of a massless rigid rod of length a. The system is free to rotate in three dimensions about the fixed center of the rod.

a) Show that the allowed energies of this rigid rotor are

$$E_n = \frac{\hbar^2 n(n+1)}{ma^2}, \quad \text{for } n = 0, 1, 2, ...$$

b) What are the normalized eigenfunctions for this system? Discuss any degeneracy.

Problem B3*: Find the bound state energy and wave function of a particle of mass m subject to an attractive δ-function potential

$$V(x) = -V_0 a \delta(x),$$

where a is a length parameter and $V_0 > 0$.

Problem B4: A quantum particle of mass m, energy E moving in one dimension strikes a rectangular barrier of height $V_0 = 8E/9$ and thickness d.

a) Determine the transmission coefficient across the barrier. Completely simplify your result.

b) What are the maximum and minimum transmission coefficients and for what values of d do they occur?

Problem B5: A particle of mass m and charge q is in a one-dimensional harmonic oscillator potential moving with frequency ω. In addition, it is subject to a *weak* electric field $\mathcal{E}$.

a) Find the exact expression for the energy.

b) Calculate the energy up to the first non-zero correction in non-degenerate perturbation theory. Compare your result to what you found in part a.

Modern Physics **Name** ______________________________

Ph.D. Qualifying Exam, Fall 2010

INSTRUCTIONS: There are nine problems on this exam: Four in section A, and five in section B. You must solve SIX problem with at least two from each section. You must also solve the problems marked with *. Do each problem on separate sheets of paper and turn in only those problems you want to have graded. Write your name and problem number on each page.

Problem A1: Find the time-dependent relativistic expressions for the position and velocity of a particle subject to a constant force mg. Assume the particle is at rest at the origin at $t = 0$. Also, determine the limiting speed of the particle. Recall that $p \neq mv$ for relativistic speeds.

Problem A2: Planck's law for blackbody radiation tells us that the frequency-temperature distribution is given by

$$u(\nu, T) = \frac{8\pi h}{C^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}.$$

a) Prove that in the limit $h\nu \ll kT$, that Planck's law gives Rayleigh-Jean's law:

$$u(\nu, T) \approx \frac{8\pi\nu^2}{C^3} kT.$$

b) Prove that in the limit $h\nu \gg kT$, that Planck's law gives Wein's law:

$$u(\nu, T) \approx D\nu^3 e^{\frac{-\beta\nu}{T}},$$

with the appropriate definitions of D and β.

Problem A3: An elliptical object described by

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1,$$

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS
UNIVERSITY PARK, MIAMI, FL33199

Ph.D. Qualifying Exam, Fall 2011 – MODERN PHYSICS

THURSDAY, AUGUST 18, 2011 1PM - 5PM

INSTRUCTIONS:

1. There are nine problems on this exam with
   - five in **SPECIAL RELATIVITY/MODERN PHYSICS**;
   - four in **QUANTUM MECHANICS**.
2. You must solve a total of **SIX** problems with
   - at least two from **SPECIAL RELATIVITY/MODERN PHYSICS**;
   - at least two from **QUANTUM MECHANICS**.
3. You **MUST** solve the problems marked **REQUIRED**.
4. Do each problem on separate sheets of paper and turn in only those problems you want to have graded.
5. Write your name and problem number on each page.

RELATIVITY / MODERN PHYSICS

1. Xenofo and Ydex are fighting a duel with identical paintball guns. Xenofo is at the origin of an Earth reference frame and Ydex is at $x = L$. An earth observer notes that they fire their guns simultaneously. An observer on a space-craft moving at high speed in the positive x direction notes that Ydex fired before Xenofo by an amount T. How fast is the space-craft moving relative to the Earth?

2. In the lab reference frame, an electron and a positron emerge from a nuclear reaction with equal speeds of βc. The electron is observed to be moving at an angle θ above the positive x axis, and the positron is moving at an angle θ below the positive x axis.

   (a) What is the velocity of a moving observer (S') who sees the two particles move outward along the positive and negative y' axis respectively.

   (b) What is the speed of each particle in S'?

3. A ball of mass m and speed βc collides completely inelastically with a ball of mass $2m$ moving in the opposite direction with speed v.

   (a) If the two balls are at rest after collision, determine v.

   (b) Thermal energy, Q, is generated by the collision, and is dissipated to the surroundings as the balls cool down to the temperature they had before the collision. Determine Q.

4. **REQUIRED** In a Stern-Gerlach experiment, silver atoms of mass m, whose spin is due to a single electron, are obtained from an oven at temperature T. Your answers to all parts should be in terms of given symbols and any other standard physical constants.

   (a) What is their root mean square (RMS) speed?

   (b) The atoms pass perpendicularly through a magnetic field of length L that has a gradient, $\frac{dB}{dz}$, perpendicular to the beam. For atoms in the two beams having the same RMS speed, what force do they experience?

   (c) How far apart will they be as they emerge from the field?

5. A radioactive nucleus decays via the α-emission reaction

$$A \to B + \alpha \tag{1}$$

with a half-life of $t_{1/2}$.

(a) Calculate the kinetic energy of the α-particle released in the rest frame of A.

(b) At time $t = 0$ a pure sample of A is prepared. At time $t = T$, the activity is measured to be R. What is the original mass of the sample?

QUANTUM MECHANICS

6. Consider two particles of mass m interacting with a potential in one dimension of the form

$$V(x_1, x_2) = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 + \alpha x_1 x_2, \tag{2}$$

where x_1 and x_2 are the locations of the two particles. Assume that $k_1, k_2, \alpha > 0$ and $k_1 k_2 > \alpha^2$.

Write down the time independent Schrödinger equation and find all the bound state energy eigenvalues.

7. Let H be a quantum mechanical Hamiltonian with eigenfunctions $|\phi_i\rangle$ and corresponding eigenvalues E_i with $i = 0, 1, 2, \cdots \infty$. Further assume that

$$E_0 < E_1 < E_2 < \cdots < E_\infty. \tag{3}$$

(a) Let $|\psi\rangle$ be a normalized wavefunction (not necessarily an eigenfunction of H). Prove that

$$\langle\psi|H|\psi\rangle \geq E_0. \tag{4}$$

Hint: Expand $|\psi\rangle$ in the eigenbasis of H.

(b) Prove that every attractive potential in one dimension, i.e., $V(x) < 0$ for all x, has at least one bound state. Hint: Consider the normalized wave function,

$$\psi_\alpha(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{1}{2}\alpha x^2}; \quad \alpha > 0 \tag{5}$$

and calculate,

$$E(\alpha) = \langle\psi_\alpha|H|\psi_\alpha\rangle; \quad H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x). \tag{6}$$

Show that $E(\alpha)$ can be made negative by a suitable choice of α.

8. Consider a one-dimensional harmonic oscillator in the ground state $|0\rangle$ at $t = -\infty$. Let a perturbation

$$H^1(t) = -Xe^{-\frac{t^2}{\tau^2}} \tag{7}$$

with X being the position operator be applied between $t = -\infty$ and $t = +\infty$. What is the probability that the oscillator is in the state $|n\rangle$ at $t = \infty$?

9. **REQUIRED** Consider the one dimensional harmonic oscillator.

(a) Prove that

$$[a, a^\dagger] = 1. \tag{8}$$

(b) Prove that

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \tag{9}$$

(c) Prove that

$$H|n\rangle = \hbar\omega\left(n + \frac{1}{2}\right)|n\rangle. \tag{10}$$

(d) Prove that

$$\langle n|T|n\rangle = \langle n|V|n\rangle \tag{11}$$

for any eigenstate $|n\rangle$.

Some useful information

- Some useful information about quantum harmonic oscillators in one dimension: Using standard notation, let ω be the classical frequency of the oscillator and let m be the mass of the particle. Let $|n\rangle$, $n = 0, 1, \cdots, \infty$ label all the bound eigenstates. The coordinate and momentum operators are given by

  $$X = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger); \quad P = i\sqrt{\frac{m\omega\hbar}{2}}(a^\dagger - a); \tag{12}$$

  with $a^\dagger$ and a being the step up and step down operators obeying

  $$a|n\rangle = \sqrt{n}|n-1\rangle; \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{13}$$

- $$\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = 1. \tag{14}$$

  for all $\alpha > 0$.

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS

UNIVERSITY PARK, MIAMI, FL33199

Ph.D. Qualifying Exam, Fall 2012 – MODERN PHYSICS

THURSDAY, AUGUST 16, 2012 1PM - 5PM

INSTRUCTIONS:

1. There are nine problems on this exam with
   - five in **SPECIAL RELATIVITY/MODERN PHYSICS**;
   - four in **QUANTUM MECHANICS**.
2. You must solve a total of **SIX** problems with
   - at least two from **SPECIAL RELATIVITY/MODERN PHYSICS**;
   - at least two from **QUANTUM MECHANICS**.
3. You **MUST** solve the problems marked **REQUIRED**.
4. Do each problem on separate sheets of paper and turn in only those problems you want to have graded.
5. Write your name and problem number on each page.

RELATIVITY / MODERN PHYSICS

1. **REQUIRED** In the lab system S, a mass $m_1 = 9$ is moving towads a target mass $m_2 = 16$, which is at rest. The energy of m_1 is $E_1 = 27.75$. Assume time-space dimension is $1+1$.

   (a) Calculate the space momentum p_1 of m_1. What is its velocity v_1?

   (b) Find the Lorentz transformation $\Lambda : S \to S'$,

   $$\Lambda = \begin{pmatrix} \cosh\xi & -\sinh\xi \\ -\sinh\xi & \cosh\xi \end{pmatrix}, \tag{1}$$

   to the center-of-mass system S', i.e. find the numbers for $\cosh\xi$ and $\sinh\xi$. What is the boost velocity v?

   (c) What are the energies E'_1 and E'_2 in the center-of-mass system?

   (d) Supposes m_1 and m_2 collide and stick together, forming one combined mass M. What is M? Compare M to $m_1 + m_2$.

   Hints: We are using units such that $c = 1$ and all masses/energies are given in some unit m_u. (For example, choosing $m_u = 931.5$MeV, the target mass could be Oxygen ${}^{16}_{8}$O and the projectile would be Beryllium ${}^{9}_{4}$Be).

2. The proper length for one spaceship is twice that of the other. To an observer on earth, both ships have the same length. If the faster spaceship is moving with $\beta = \frac{v}{c} = 0.95$

   (a) Find the speed of the slower ship.

   (b) If both ships are moving in the same direction, find the speed of the slower ship in the rest frame of the faster ship.

   Hint: Make sure that the *sign* of the velocity you find makes sense.

3. An observer in a rocket moves toward a mirror at speed v relative to the mirror. A light emitted by the rocket moves toward the mirror and is reflected back to the rocket. If the rocket is a distance d away from the mirror when it emits the light pulse as measured in the mirror's reference frame, what is the total travel time of the pulse (from the rocket and back) as measured in

   (a) the rest frace of the mirror and

(b) the rest frame of the rocket?

4. The activity (decays per second), R, of a radioactive sample decreases to $\frac{R}{32}$ in time T.

   (a) What is the half-life and decay constant?

   (b) What fraction of the original number of nuclei will have decayed after $\frac{T}{100}$ and $\frac{T}{10}$.

5. The radial part of the wavefunction for the Hydrogen atom in the $2p$ state is given by

$$R_{2p}(r) = Are^{-\frac{r}{2a_0}}, \tag{2}$$

   where A is a constant and a_0 is the Bohr radius. Use this to calculate the average values of r, r^2 and the standard error in r.

QUANTUM MECHANICS

6. **REQUIRED** Eight electrons are placed in a three-dimensional infinite cubic well with sides of length L.

   (a) Determine the ground state energy.

   (b) Determine the energy of the first excited state.

7. This problem involves the case of a harmonic oscillator in the ground state ($n = 1$). In some cases you may use physics arguments instead of direct calculation to find the answers.

   (a) Find $\langle x \rangle$ and $\langle x^2 \rangle$.

   (b) Find $\langle p \rangle$ and $\langle p^2 \rangle$.

   (c) Use your results from the previous two parts to determine $\Delta x \Delta p$.

   (d) What is special about this case and why?

8. Consider a quantum mechanical particle of mass m constrained to move in a one dimension. Let x label the space variable and let $x \in [0, L]$. Also assume periodic boundary conditions: $x = 0$ is identified with $x = L$. As such, you can also visualize space as a circle with perimeter L. Let this particle interact with a potential $V(x)$ which satifies the property

$$V(x + a) = V(x) \tag{3}$$

for all x. In addition a satisfies the relation $Na = L$ with N being an integer. Assuming no degeneracies in the spectrum of bound states, prove that all eigenfunctions satisfy the condition

$$\psi(x + a) = C\psi(x) \tag{4}$$

with C being one of the N'th root of unity, i.e; $C^N = 1$.

9. Consider Hermitian matrices γ_i, $i = 1, 2, 3, 4$ that obey

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta_{ij}I; \quad i, j, = 1, 2, 3, 4. \tag{5}$$

The Kronecker delta is defined as

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} . \tag{6}$$

(a) Show that the eigenvalues of γ_i are ± 1 for all i. Hint: Go to the eigenbasis of γ_i, and use the equation for $i = j$.

(b) By considering the relation (5) for $i \neq j$ and noting that trace is cyclic, namely; $\mathrm{Tr}(ACB) = \mathrm{Tr}(CBA)$, show that γ_i is traceless for all i.

(c) Show that γ_i cannot be odd dimensional matrices.

Some useful information

- The ground state eigenfunction of a harmonic oscillator is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}}. \tag{7}$$

- $$\sqrt{\frac{\alpha}{\pi}} \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = 1. \tag{8}$$

  for all $\alpha > 0$.

FLORIDA INTERNATIONAL UNIVERSITY

DEPARTMENT OF PHYSICS

UNIVERSITY PARK, MIAMI, FL 33199

INSTRUCTIONS

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- four in SPECIAL RELATIVITY/MODERN PHYSICS;
- five in QUANTUM MECHANICS.

You must solve a total of **SIX** problems with

- at least two from SPECIAL RELATIVITY/MODERN PHYSICS;
- at least two from QUANTUM MECHANICS.

You **MUST** solve the problems marked as **REQUIRED**.

Do each problem on separate sheets of paper and turn in only those problems you want to have graded.

Write your assigned number **(DO NOT WRITE YOUR NAME)** and problem number on each page.

1. (Special Relativity/Modern Physics)

Derive the relativistic explanation of superluminal motion.

Superluminal motion is the apparent faster-than-light motion seen in some radiogalaxies and other sources. Consider an active galactic nucleus (A) that emits a relativistic jet from its center at time t_1. This jet has a speed v in a direction that makes an angle θ with respect to the line joining the observer on earth (O) and the origin of the relativistic jet at time t_1 (A). Light emitted from this jet is seen by this observer as a streak in the sky.

(a) Obtain an expression for the apparent transverse speed of this streak as a function of v, θ and the speed of light c in the limit where the distance OA is very very large.

(b) Find the maximum value of the apparent transverse speed for a fixed v.

(c) Show that this maximum value can be larger than c even though v has to be less than or equal to c.

2. (Special Relativity/Modern Physics)

An Imperial Star Destroyer passes a rebel observation post at a relativistic speed. The observation post's sensors measure the length of the Star Destroyer to be 800m. (You can assume that this is an instantaneous measurement of the length).

(a) Assuming that the length of the Star Destroyer is 1,600 m at rest, how fast is the ship traveling relative to the observation post? Leave your answer in terms of c, the speed of light.

(b) In response to the approaching Star Destroyer, the observation post sends an alert signal to the rebels on a nearby planet. If the signal travels at the speed of light, how fast does this signal travel according to the observer on the Star Destroyer? Explain your answer.

(c) As the Star Destroyer passes and moves away from the post, the post's sensors monitor Darth Vader in his private gymnasium on the Star Destroyer practicing his light saber skills. If the sensors measure infrared photons from the light saber with $\lambda = 2500$ nm, determine the wavelength of light that Darth Vader observes.

3. (Special Relativity/Modern Physics) **<u>REQUIRED</u>**

When a muon decays at rest,

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

why can the electron only have a maximum energy of 53 MeV? The mass of the muon is 106 MeV/c^2 and you can assume that the neutrinos are massless. You can also ignore the mass of the electron.

4. (Special Relativity/Modern Physics)

A large number of hydrogen atoms are in the n = 3 state.

(a) Considering all possible transitions that the atoms can make before reaching their ground state, what wavelengths would you observe in the emission spectrum?

(b) If one such atom, initially at rest, undergoes a transition <u>directly</u> from the n = 3 state to the ground state,

i. what is the momentum of the emitted photon, and

ii. with what kinetic energy will the atom recoil?

5. (Quantum Mechanics)

Consider three spin 1/2 particles, each bound in the 1-dimensional harmonic oscillator potential. The single particle state vectors are given by $|n,m>_1$, $|n,m>_2$ and $|n,m>_3$, where n is the energy state number (n=0,1,2,3,...) and m is the z-component of the spin (m = ±1/2).

(a) Assume that the particles have equal mass, but are distinguishable.
   i. Determine the ground state energy of this system of distinguishable particles.
   ii. Determine the ground state vector(s) as a product of the single particle state vectors. What is the degeneracy of the ground state?

(b) Now assume that the particles are identical fermions.
   i. Determine the ground state energy of this system of identical particles.
   ii. Determine the ground state vector(s) as a product of the single particle state vectors. What is the degeneracy of the ground state?

6. (Quantum Mechanics)

A hydrogen atom is in the ground state. A pulsed electric field $E(t) = E_0\delta(t)$ is applied to the atom. Calculate the transition probability to the excited state n with the energy E_n and the eigenfunction ϕ_{nlm} ($n, l,$ and $m,$ are quantum numbers) and calculate the probability of the atom remaining in the ground state.

7. (Quantum Mechanics) **REQUIRED**

A particle with mass m is in the *n'th* bound state (with the energy E_n and the wavefunction $\phi_n(x)$) in a 1-D square potential well (the potential depth is V_0 and the width is w). Find

(a) the probability of the particle outside the potential well and

(b) the expectation values of $V(x)$ and $V^2(x)$ under the condition that $V_0 >> E_n$.

8. (Quantum Mechanics)

An operator **A**, representing observable A, has two normalized eigenstates ψ_1 and ψ_2 with eigenvalues a_1 and a_2, respectively. Operator **B**, representing observable B, has two normalized eigenstates ϕ_1 and ϕ_2 with eigenvalues b_1 and b_2, respectively. The eigenstates are related by $\psi_1 = (\sqrt{3}\,\phi_1 - \phi_2)/2$ and $\psi_2 = (\phi_1 + \sqrt{3}\,\phi_2)/2$.

(a) Observable *A* is measured, and the value a_2 is obtained. What is the state of the system (immediately) after this measurement?

(b) If *B* is now measured, what are the possible results and what are their probabilities?

(c) Right after the measurement of *B*, *A* is measured again. What is the probability of getting a_2? [Note that the answer would be quite different if I had told you the outcome of the *B* measurement.]

9. (Quantum Mechanics)

Find the differential cross section for elastic scattering of a particle initially traveling along the z-axis from a non-spherical, double delta potential

$$V(\vec{r}) = V_0\delta(\vec{r} - a\hat{z}) + V_0\delta(\vec{r} + a\hat{z})$$

where $\hat{z}$ is a unit vector along the z-axis.

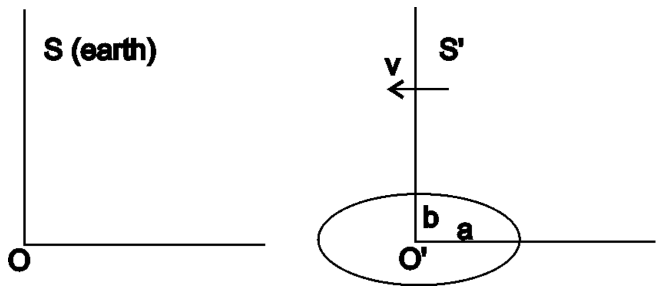
(a) Since the potential is not spherically symmetric, obtain the scattering amplitude from the general equation

$$f(\vec{q}) = -\frac{m}{2\pi\hbar^2}\int e^{i\vec{q}\cdot\vec{r}_0}V(\vec{r}_0)d^3\vec{r}_0$$

where $\vec{q} = \vec{k}' - \vec{k}$ is the momentum transfer.

(b) Since the incident particle is initially traveling along the z-axis, and it scatters elastically from the potential, the magnitudes of the momenta are the same before and after the collision. Let θ be the angle of the outgoing momentum with respect to the z-axis. Find the differential cross section $d\sigma/d\Omega = |f(\theta)|^2$ in terms of constants and the magnitude of the momentum, k.

with $a > b$, is moving towards earth in a direction parallel with the x' axis and with a speed v relative to the earth. To earth observers, the semi-major axis is contracted to the same length, a, as the semi-minor axis, b.



a) Determine an expression for v.

b) Show that to earth observers, the object is a perfect circle of radius b. Hint: Consider the instant, $t = t' = 0$, when O and O' coincide, transform an arbitrary point (x', y') on the ellipse to the earth reference frame, and determine the equation for the object's shape.

Problem A4: A stationary particle of mass M absorbs a high energy photon and breaks into two particles, each of mass m, that move away from the impact point with equal speeds of βc, and at equal angles of θ relative to the direction of the photon.

a) In terms of m and c, what was the energy of the photon?

b) In terms of m, what was the mass, M, of the stationary particle?

Problem B1: Quarks carry spin 1/2. Three quarks bind together to make a baryon (such as udd in a neutron and uud in a proton). Assume the quarks are in the ground state (so the orbital angular momentum is zero). What spins are possible for baryons? If one further assumes that there is no spin-spin interaction, what fraction of the baryons will be in total spin 3/2 state?

Problem B2: Consider 1D finite square well potential

$$V(x) = \begin{cases} -V_0 & -a < x < a \\ 0 & |a| > a \end{cases}$$

where $V_0 > 0$. Show that there is always at least one bound state for a particle of mass m. Find the relationship between m, V_0, a and $\hbar$ that will give a bound (meaning negative energy) first excited state.

Problem B3: In a quantum system, the eigenstates and eigenvalues of a particle are given by ϕ_n and E_n (n=0, 1,2...). When $t \leq 0$ the particle is in the ground state ϕ_0. At $t > 0$, a small perturbation $H'(x,t) = F(x)\exp(-t/\tau)$ is turned on.

What are the probabilities for the particle to be in the excited state ϕ_n after $t > \tau$ (use the first-order perturbation approximation).

Problem B4: A particle with mass m moves in a central potential

$$V(r) = \frac{\alpha}{r^s} \quad \text{for } \alpha > 0.$$

Show (a) the condition for the existence of bound states is $0 < s < 2$ (or is it $0 < s \leq 2$?) and (b) there are infinite number of bound states near $E \sim 0^-$ (should be $E \sim 0^+$?).

Problem B5*: Two identical particles are in a 1-D harmonic potential

$$V(x) = \frac{1}{2} m\omega^2 x^2.$$

The mutual interaction between the two particles is given by

$$V_{int}(x_1, x_2) = V_0 e^{-\beta^2 (x_1 - x_2)^2}.$$

Find the ground-state energy of the two-particle system, including the first-order correction by treating V_{int} as a small perturbation.

Modern Physics
Ph.D. Qualifying Exam   Fall 2014
Florida International University Department of Physics

Instructions: There are nine problems on this exam. Five on quantum mechanics (Section A), and four on general modern physics (Section B). You must solve a total of six problems with at least two from each section. You must also do the problems marked **Required**.

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You may use a calculator and the math handbook as needed.

Section A

1. A particle is moving inside a 1-D potential $V(x) = \infty$ for $x<0$; $V(x) = \frac{\mu\omega^2 x^2}{2}$ for $x>0$. Find the particle energy.

2. A particle of mass m moves between two infinite potential walls separated by a distance d.
   a) Find the ground state energy.
   b) One of the potential walls is suddenly and instantaneously moved to a distance $2d$. What is the probability of finding the particle in the new ground state?

3. An electron with magnetic moment μ_0 and spin up at t=0 enters a region of a static magnetic field $H = -H_0\hat{z}$. There is also a AC magnetic field given by $H' = H_1\cos(\omega t)\hat{x} + H_1\cos(\omega t)\hat{y}$. What is the probability of finding the electron in the spin up state at time t?

4. A particle with positive energy ($E > 0$) encounters a Dirac delta-function potential barrier

$$V(x) = \alpha\delta(x)$$

where α is positive real constant with the units of energy×length.
   a) Solve the Schrödinger equation for the regions $x < 0$ and $x > 0$. If you make any assumptions, you must verify your answer to receive full credit.
   b) State and apply the appropriate boundary conditions for the potential interface at $x = 0$. Obtain expressions relating the different coefficients.
   c) Using the results from part b), determine the transmission coefficient T for a particle incident and transmitted through this potential barrier. Discuss the limiting cases for T as the energy E approaches zero ($E \to 0$) and E approaches infinity ($E \to \infty$).

5. **Required** Consider an unperturbed harmonic oscillator with Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2,$$

with eigenstates of the form $|n\rangle$, and energy eigenvalues

$$E_n^0 = \left(n + \frac{1}{2}\right)\hbar\omega.$$

A perturbation of the form

$$H' = \frac{1}{2}\beta m\omega^2 x^2$$

is added to the system, with β being a small parameter.

a) Using the definition for the harmonic oscillator raising and lowering operators, show that the expectation value of x^2 can be written as

$$\langle n|x^2|n'\rangle = \frac{\hbar}{2m\omega}\left(\sqrt{(n'+1)(n'+2)}\,\delta_{n,n'+2} + \sqrt{n'(n'-1)}\,\delta_{n,n'-2} + (2n'+1)\delta_{n,n'}\right)$$

n'

b) Using non-degenerate perturbation theory, obtain the first-order correction to the energy.

c) Using non-degenerate perturbation theory, obtain the second-order correction to the energy. You may only consider the n = 0 and n = 1 states when doing the expansion.

Hint: Recall the following relationships

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x) \qquad a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-) \qquad a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

Section B

6. Light of wavelength 300 nm strikes a metal plate, producing photoelectrons that move with a speed of $0.002c$.

   a) Determine the work function of the metal.
   b) What is the critical wavelength for this metal, so that photoelectrons are produced?
   c) What is the physical significance of the critical wavelength?

7. Provide a brief qualitative description for each physical phenomenon:

   a) Compton Effect.
   b) Difference between bosons and fermions.
   c) Michelson Morley Experiment.
   d) Rutherford scattering.

8. **Required** A pion, π^-, has a rest mass of 139.57 MeV/c and is moving with a speed $\beta = 0.5c$. It then decays into a muon/neutrino pair, $\mu^-\nu_\mu$, with the muon traveling off at an angle of 10^0 relative to the original direction of the pion. What is the energy and momentum of the muon and what is the angle, energy, and momentum of the neutrino? The muon has a rest mass of 105.65 MeV/c and you may assume the neutrino is massless

9. Police radar is used to measure the speed of oncoming cars. It works by broadcasting microwaves of a precisely known frequency, *f*. The moving car reflects these waves that are then picked up by the radar gun. The gun computes the speed of the car by measuring the beat frequency (which is small compared to the broadcast frequency) between the broadcast frequency and the detected frequency.

   a) Find the frequency of the reflected wave in terms of *f*, *v*, and *c.*
   b) Show that the speed of the car is given by $v =\cong \frac{1}{2}\frac{f_{beat}}{f}c$ for a car moving much less than the speed of light.

Modern Physics
Ph.D. Qualifying Exam   Fall 2015
Florida International University Department of Physics

Instructions: There are nine problems on this exam. Six on quantum mechanics (Section A), and three on general modern physics (Section B). You must solve a total of six problems with at least two from each section. You must also do the problems marked **Required**.

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Section A

1. An electron (spin ½) at rest is in a uniform magnetic field $\vec{B} = B_y\hat{j} + B_z\hat{k}$ (B_y>>B_z). The Hamiltonian is $H = H_0 + H' = \frac{eB_y}{m}S_y + \frac{eB_z}{m}S_z$ (*m* is the electron mass and e is the electron charge). Find the first-order perturbation corrections of the electron spin wave functions.

2. (**Required**) A particle of mass *m* is confined to move freely in a ring of radius *R*.
(a) Find the energies and the eigenfunctions of the particle.
(b) A perturbation term

$$H' = \begin{cases} V_1, -\alpha < \varphi < 0 \\ V_2,\ 0 < \varphi < \alpha \\ 0, \quad \text{elsewhere} \end{cases}$$

is added to the particle, where ϕ is the azimuthal angle and α is an arbitrary fixed value. Find the first order corrections of the energies for the three lowest energy states.

3. An electron in a hydrogen atom jumps from $n = 4$ to $n = 1$ state.
(a) Is energy absorbed or emitted in this process? If energy is emitted, which spectral line series is this transition? Explain.
(b) Determine the energy transfer in eV and photon wavelength in nm for this process.
(c) Explain why classical physics fails to correctly describe the emission spectrum for the hydrogen atom.
(d) Discuss Bohr's model of the hydrogen atom. In your discussion, be sure to include his three revolutionary postulates that led to this model.

4. A 1-D particle in an infinite square well $(0 \le x \le a)$ has the initial wavefunction $\Psi(x,0) = Ax(a-x)$ inside the well and $\Psi(x,0) = 0$ outside the well.
(a) Find the normalization constant A.
(b) Find the expectation value for the position x.
(c) Find the expectation value for x^2.
(d) Use the results of (a) and (b), find the standard deviation σ_x.
(e) If $\sigma_p = \frac{\hbar\sqrt{10}}{a}$, is the Heisenberg Uncertainty Principle satisfied?

5. An operator $\hat{A}$, representing observable A, has two normalized eigenstates Ψ_1 and Ψ_2 with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two normalized eigenstates Φ_1 and Φ_2 with eigenvalues b_1 and b_2, respectively. The eigenstates are related by $\Psi_1 = \frac{\sqrt{3}\Phi_1 - \Phi_2}{2}$, and $\Psi_2 = \frac{\Phi_1 + \sqrt{3}\Phi_2}{2}$.
(a) Observable A is measured, and the value of a_2 is obtained. What is the state of the system immediately after this measurement?
(b) If B is now measured, what are the possible results and what are their probabilities?
(c) Right after the measurement of B, A is measured again. What is the probability of getting a_2?

6. There are three particles and four distinct single-particle states Φ_1, Φ_2, Φ_3, and Φ_4. How many different three-particle states can be formed, (a) if the three particles are distinguishable, (b) if the three particles are identical fermions, and (d) the three particles are identical bosons?

Section B

7. In a nuclear physics experiment, a photon beam with the energy of E_γ is incident on a fixed proton target (liquid hydrogen). Suppose you are interested in production of the strange particle Λ, via the reaction of $\gamma + p \rightarrow K^+ + \Lambda$, what is the minimum E_γ you need to produce the Λ? The masses of the proton, K^+, and Λ are 0.938, 0.494 and 1.116 GeV, respectively.

8. (**Required**) Consider the process of Compton scattering. A photon of wavelength λ is scattered off a free electron initially at rest. Let λ' be the wavelength of the photon scattered in a direction θ relative to the photon incident direction.
(a) Find λ' in terms of λ and θ and universal constants.
(b) Find the kinetic energy of the recoiled electron.

9. (a) A laser emits a pulse of light which travels at a speed of c (in vacuum) relative to the laser. Does this mean that the speed of the laser relative to the light pulse is also c? Give your reasoning for your answer.

(b) A laser emits a pulse of light in the positive *y* direction at the same time that a space-ship passes the laser at a speed of 0.9c in the positive *x* direction. What is the speed and direction of the light pulse relative to the space-ship?