Homework 1 - MAS 5145 Summer A - 2013 **NAME:** ________________________________

To receive credit you MUST SHOW ALL YOUR WORK.

1. (8 pts) For both parts of the problem, let $A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$ and do the change of variables

$$y_1 = 3x_1 - 2x_2, \quad y_2 = x_1 + x_2.$$

(a) Solve $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}, \quad \text{where } \mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$

That is, express $x_1(t)$ and $x_2(t)$ in terms of the initial conditions $x_1(0), x_2(0)$.

(b) Solve $\dfrac{d^2\mathbf{x}}{dt^2} = -A\mathbf{x}.$

That is, express $x_1(t)$ and $x_2(t)$ in terms of the initial conditions $x_1(0), x_2(0), x_1'(0), x_2'(0)$.

2. (8 pts) Let A be an arbitrary set. Denote by $(\mathbb{R}^A)_0$ the set of all functions $f : A \to \mathbb{R}$ with finite support, that is, $f \in (\mathbb{R}^A)_0$ if and only if $f(a) \neq 0$ for only finitely many elements $a \in A$.

(a) (4 pts) Show that $(\mathbb{R}^A)_0$ is a real vector space which has as basis a set which is bijective to A. (For this reason, $(\mathbb{R}^A)_0$ is called the vector space generated by A. You thus prove that there are vector spaces with basis of arbitrary cardinality.)

(b) (4 pts) If $A = \mathbb{N}$, the set of natural numbers, prove that the set of real sequences with finite support $(\mathbb{R}^{\mathbb{N}})_0$ is isomorphic with the space of polynomials with real coefficients $\mathbb{R}[t]$.

(The "0" subscript is very important and was forgotten in the lecture. $\mathbb{R}[t]$ and $(\mathbb{R}^{\mathbb{N}})_0$ are **not** isomorphic with the set all sequences $\mathbb{R}^{\mathbb{N}}$. You'll receive 2 bonus points if you correctly justify this.)

3. (4 pts) Pb. 12, page 21, textbook. Which of the following sets of vectors are linearly independent? Which span? Which are bases?

$$\text{In } \mathbb{R}^3, \text{ the columns of } \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 1 & 2 & 5 \\ 3 & 3 & 4 & 8 \end{pmatrix}$$