**Homework 2 - MAS 5145** Summer A 2013 **NAME:** ____________________

**1.** (16 pts) A linear operator $p : V \to V$ is called a *projector* of the vector space $V$ if $p^2 = p$. We denote $p^2 = p \circ p$. Show that if $p$ is a projector of $V$, then:

(a) $V = \mathrm{Im}p \oplus \mathrm{Ker}p$ ;

(b) the operator $q = Id_V - p$ is also a projector of $V$ ($Id_V$ denotes the identity of $V$), and $\mathrm{Ker}q = \mathrm{Im}p$, $\mathrm{Ker}p = \mathrm{Im}q$;

(c) the operator $s = 2p - Id_V$ is an involutive automorphism of $V$; that is, you should show that $s^2 = Id_V$ and that $s$ is an isomorphism from $V$ to $V$.

(d) Let $V = M_{n,n}(\mathbf{R})$ be the vector space of real $n \times n$ matrices and let $p : V \to V$ be the operator $p(A) = \frac{1}{2}(A - A^T)$, where $A^T$ denotes the transpose of the matrix $A$. Show that $p$ is a projector and describe in words the subspaces $\mathrm{Ker}p$ and $\mathrm{Im}p$. Describe bases for them and find their dimensions.

**2.** (4 pts) Pb. 7, p. 56 textbook. Consider the operator $L : \mathbb{R}_3[t] \to \mathbb{R}^2$, $L(p(t)) = (p(0), p(1))^T$. Show that $L$ is onto. What is the dimension of $\mathrm{Ker}L$.