

Exam 3- MAC 2311, Sp. 2013 **NAME:** _____ **Panther ID:** _____

General Directions: *Read the problems carefully and provide answers exactly to what is requested. Use complete sentences and use notation correctly. Incomprehensible work is worthless. I am grading the work, not just the answer. Don't rush, don't try to do too many steps of a computation at once; work carefully. Good luck!*

1. (8 pts) Solve the initial value problem:

$$\frac{dy}{dx} = 3e^x + \frac{\cos x}{2} - 1, \quad y(0) = 1$$

2. (18 pts) Find each of the following limits:

(a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\ln(3x + 1)}$

(b) $\lim_{x \rightarrow 0^+} (\sin x)^x$

3. (16 pts) Find each indicated antiderivative:

(a) $\int x^2 \sqrt{x^3 + 9} dx$

(b) $\int \frac{1}{x(\ln x)^2} dx$

4. (14 pts) A rectangle has its two lower corners on the x -axis and its two upper corners on the curve $y = 36 - x^2$. For all such rectangles, what are the dimensions of the one with largest area.

5. (12 pts) Find the absolute minimum and the absolute maximum (if they exist) for the function $f(x) = 4x + \frac{1}{x}$ over the interval $(0, 1]$.

6. (12 pts) (a) (8 pts) Find the local linear approximation of the function $f(x) = \sqrt[3]{x}$ at $x = 8$.

(b) (4 pts) Use part (a) to approximate $\sqrt[3]{7.94}$ without using a calculator.

7. (12 pts) A rocket that is launched vertically is tracked by a radar station located on the ground 8 miles from the launch site. What is the vertical speed of the rocket at the instant its distance from the radar station is 10 miles and this distance increases at the rate of 2400 mi/h?

8. (18 pts) Give a complete graph of the function $f(x) = x^2e^{-2x}$. Your work should include a sign chart for the derivative and the second derivative, the coordinates of the critical points and the end-behavior of the function. Determine also the concavity of the function and x -coordinates of the inflection points (if any). To save you time, I give you the second derivative $f''(x) = 2(2x^2 - 4x + 2)e^{-2x}$.