Prerequisite Test – Calculus 1 - Fall 2014 NAME: Solution Key / Grading Key

1. (6 pts) Fill in the exact values: (No partial credit), 1pt each)

$\cos(\frac{\pi}{4}) = \boxed{\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}}$ $\sin(\frac{7\pi}{2}) = \sin\left(\frac{3\pi}{2} + 2\pi\right) = \boxed{-1}$ $\sin^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\frac{\pi}{3}}$

$\left(\frac{4}{9}\right)^{-1/2} = \frac{1}{\sqrt{\frac{4}{9}}} = \frac{1}{\frac{2}{3}} = \boxed{\frac{3}{2}}$ $\log_2 32 = \log_2(2^5) = \boxed{5}$ $\log_{10}(0.001) = \log_{10}(10^{-3}) = \boxed{-3}$

2. (4 pts) Circle the correct answer (assume that $x \neq 0$): 2pts each ; no partial credit

(a) The expression $\frac{2x^2}{x^4+2x^2}$ is equivalent with:

$\frac{2x^2}{x^4+2x^2} = \frac{2x^2}{x^2(x^2+2)} = \frac{2}{x^2+2}$

(i) $\frac{1}{x^4+1}$ (ii) $\frac{2}{x^2}+1$ (iii) $\frac{2}{x^2+2}$ (iv) $\frac{1}{x^2+1}$ (v) $\frac{2}{3x^2}$

(b) The expression $\frac{x^2}{\sqrt[3]{x^2}}$ is equivalent with:

$\frac{x^2}{\sqrt[3]{x^2}} = \frac{x^2}{x^{\frac{2}{3}}} = x^{2-\frac{2}{3}} = x^{\frac{4}{3}} = x\sqrt[3]{x}$

(i) $\sqrt{x}$ (ii) 1 (iii) $x\sqrt[3]{x}$ (iv) $x^{-1/3}$ (v) none of the above

3. (4 pts) Find the domain of each of the following functions. Write your answer in interval form.

(a) $f(x) = \sqrt{x+1} - \sqrt{6-2x}$

(1pt) $x+1 \geq 0$ and $6-2x \geq 0$

(0.5pts) $x \geq -1$ and $x \leq 3$

(0.5pts) $x \in [-1, 3]$

(b) $g(x) = \frac{x}{x^2+x-6} = \frac{x}{(x+3)(x-2)}$ (1pt)

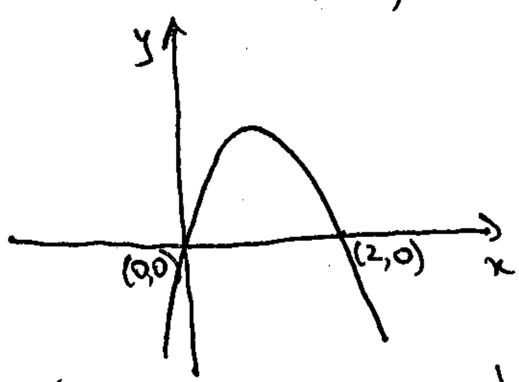
$x \neq -3$, $x \neq 2$ (0.5pts)

$x \in (-\infty, -3) \cup (-3, 2) \cup (2, +\infty)$

(0.5pts

4. (6 pts) Sketch the graph of each of the following functions and mark the coordinates of axis intercepts.

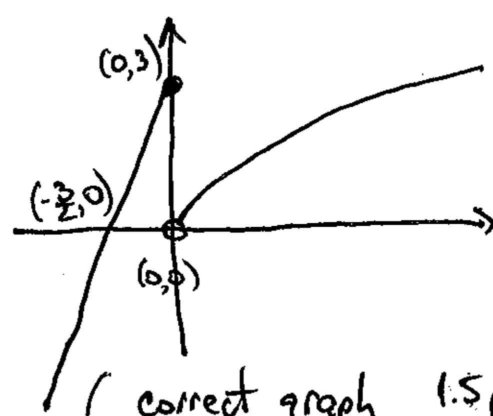
(a) $f(x) = 2x - x^2 = x(2-x)$




(correct graph 1.5pts)

(x-intercepts 0.5pts)

(b) $g(x) = \begin{cases} 2x+3 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$




(correct graph 1.5pts

x-intercepts 0.5pts)

5. (4 pts) True or False? Assume $x, y$ are positive real numbers. Circle "True" if the equality holds for all $x, y$. Otherwise, circle "False".

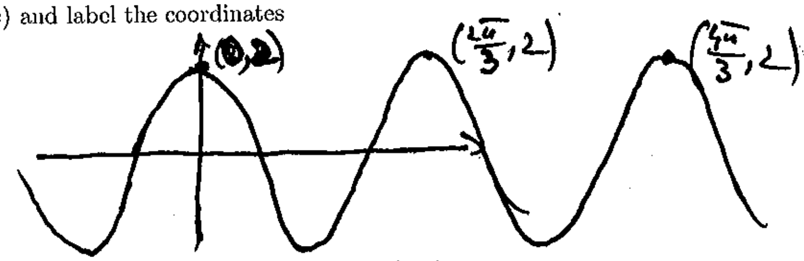
(No partial credit)

| | |
|---|---|
| $\sqrt{x^2+y^2} = x+y$ | True **False** |
| $(x+y)^{-1} = x^{-1}+y^{-1}$ | True **False** |
| $\log(x^2+y^2) = 2\log x + 2\log y$ | True **False** |
| $\sin(\frac{\pi}{2}-x-y) = \cos(x+y)$ | **True** False |

6. (3 pts) Sketch the graph of $y = 2\cos(3x)$ and label the coordinates of at least two of the maximum points (that is, points where $y$ is maximum).

(2 pts graph)

(1 pt two maxima)




$\cos(3x) = 1$

$3x = 2k\pi, \ k \in \mathbb{Z}$

$x = \frac{2k\pi}{3}, \ k \in \mathbb{Z}$

7. (3 pts) Write an equation of the line that contains the points $(-2, -13)$ and $(1, 2)$.

$$m = \frac{\Delta y}{\Delta x} = \underbrace{\frac{y_2 - y_1}{x_2 - x_1}}_{(1pt)} = \frac{2-(-13)}{1-(-2)} = \frac{15}{3} = 5$$

(0.5 pts)

Point-slope $y - y_1 = m(x - x_1)$ (1pt) $\boxed{y - 2 = 5(x-1)}$ or $\boxed{y = 5x - 3}$ (0.5 pts)

8. (6 pts) Compute and simplify the following expressions:

(a) $\dfrac{f(3+h) - f(3)}{h}$ if $f(x) = 2x - x^2$

$$\frac{f(3+h) - f(3)}{h} \underset{(1pt)}{=} \frac{(2(3+h) - (3+h)^2) - (2\cdot 3 - 3^2)}{h}$$

$$= \frac{(6+2h) - (9+6h+h^2) - 6 + 9}{h}$$

$$= \frac{\cancel{6} + 2h - \cancel{9} - 6h - h^2 \cancel{-6} \cancel{+9}}{h}$$

$$\underset{(1pt)}{=} \frac{-4h - h^2}{h} = \frac{-\cancel{h}(4+h)}{\cancel{h}} = \boxed{-4-h}$$

(1pt)

(b) $\dfrac{g(x) - g(a)}{x - a}$ if $g(x) = \dfrac{2}{1+x}$

$$\frac{g(x) - g(a)}{x - a} \underset{(0.5pts)}{=} \frac{\frac{2}{1+x} - \frac{2}{1+a}}{x-a} =$$

$$\underset{(1pt)}{=} \frac{\frac{2(1+a) - 2(1+x)}{(1+x)(1+a)}}{x-a} = \frac{\frac{\cancel{2} + 2a \cancel{-2} - 2x}{(1+x)(1+a)}}{\frac{x-a}{1}}$$

$$= \frac{-2\cancel{(x-a)}}{(1+x)(1+a)} \cdot \frac{1}{\cancel{x-a}}$$

$$\underset{(1.5pts)}{=} \frac{-2}{(1+x)(1+a)}$$

**9.** (8 pts) Find all real solutions of the following equations (2 pts each):

(a) $x^4 - 5x^2 + 4 = 0$

$(x^2-1)(x^2-4) = 0$ (1pt) $\quad x=1, x=-1, x=2, x=-2$ (1pt)

$(x-1)(x+1)(x-2)(x+2) = 0$ $\quad$ $-0.5$ for ommitting the negative roots.

(b) $3\cdot(5^{2x}) = 7$ $\div 3$ $\quad$ Leave your answer as a logarithm for this one.

$5^{2x} = \frac{7}{3}$ ← Apply ($\log$ or $\ln$ or $\log_5$) (1pt up to here)

$\ln(5^{2x}) = \ln\left(\frac{7}{3}\right) \Rightarrow 2x \ln 5 = \ln\left(\frac{7}{3}\right) \Rightarrow \boxed{x = \frac{\ln\left(\frac{7}{3}\right)}{2\ln 5}}$ or alternative forms (1pt)

(c) $\sin^2 x = \cos^2 x$ $\quad$ OK to find all solutions $x \in [0, 2\pi]$ for this one.

$\tan^2 x = 1$ (1pt) (Other ways to start are possible)

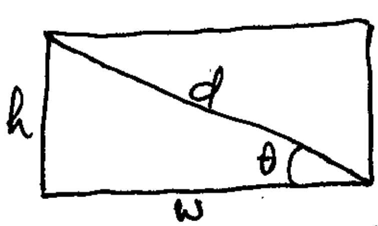
$\tan x = \pm 1$ $\quad x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}, x = \frac{7\pi}{4}$ (1pt) (−0.5pts for missing some solut

(d) $ax^2 + bx + c = 0$ $\quad$ I want to check you know the quadratic formula.

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (No partial credit) $\quad$ Note: Solutions are real when $b^2 - 4ac \geq 0$.

**10.** (6 pts) For a 16:9 TV, the ratio (width of screen)/(height of screen) is 16/9.

(a) For a 16:9 TV, find a function expressing the area of the screen, $A$, in terms of its diagonal length $d$.




Since $\frac{w}{h} = \frac{16}{9}$ can let $w = 16x$, $h = 9x$ where $x$ is a variable determined by the size of the T

Then $A = w \cdot h = 16x \cdot 9x = 144x^2$

(b) For a 16:9 TV, what is the angle that the diagonal is making with the horizontal? Leave your answer as an inverse trigonometric function.

From Pythagora

$h^2 + w^2 = d^2$ so $d^2 = (9x)^2 + (16x)^2 = 81x^2 + 256x^2 = 337x^2$

so $x^2 = \frac{d^2}{337}$ and plugging this in in $A = 144x^2$

get $\boxed{A = \frac{144}{337} d^2}$

(b) $\tan\theta = \frac{9}{16}$ (2pts) $\quad \theta = \arctan\left(\frac{9}{16}\right)$ (1pt)

or other ways.