Name: _____ **Panther ID:** _____

Exam 1 - MAC2311 - Fall 2014

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (12 pts) These are True or False questions. No justification required. No partial credit. 2 points each.

(i) For all $a > 0, b > 0$, $\log(a^2b^3) = 2\log a + 3\log b$ **True** **False**

(ii) For all $a > 0$, $\sqrt{4a^2+1} = 2a+1$ **True** **False**

(iii) For all $x \neq 0$ $\dfrac{x}{\sqrt[3]{x^2}} = \dfrac{1}{\sqrt[3]{x}}$ **True** **False**

(iv) For all $x \neq 0$, $\dfrac{\sin x}{x} = 1$ **True** **False**

(v) The function $f(x) = \tan x$ is defined and is continuous for all real numbers x. **True** **False**

(vi) If $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = -2$ then $\lim_{x\to 3}(f(x)+2g(x)) = 0$ **True** **False**

2. (6 pts) Consider the function $f(x) = \frac{1}{\sqrt{6-2x}}$.

(a) (3 pts) Find the domain of f. Write your answer in interval form.

(b) (3 pts) Compute and simplify $f(3-2a^2)$.

3. (6 pts) Find an equivalent expression, without inverse trigonometric functions, for $\sec(\arctan x)$.

4. (12 pts) An object is thrown straight up in the air from the ground. Its position $s(t)$ in feet above the ground after t seconds is given by $s(t) = 48t - 16t^2$.

(a) (3 pts) When is the object back on the ground?

(b) (3 pts) Find the average velocity of the object during the first two seconds of its flight.

(c) (6 pts) Use limits to find the instantaneous velocity of the object at 2 seconds.

5. (12 pts) Given the function below

$$g(x) = \begin{cases} kx^2 - 1 & \text{if } x \leq 1 \\ 2x + k & \text{if } x > 1 \end{cases}$$

(a) (6 pts) Is there a value of the constant k which will make the function continuous? Justify your answer.

(b) (6 pts) Sketch the graph of the function $g(x)$ when $k = 1$.
Label carefully the coordinates of important points.

6. (30 pts) Find the following limits (5 pts each). If the limit is infinite or does not exist, specify so.

(a) $\lim_{x\to 1} \dfrac{3x-3}{x^2+2x-3}$

(b) $\lim_{x\to 5^+} \dfrac{1-x}{x-5}$

(c) $\lim_{t\to 2} \dfrac{|t-2|}{t^2-4}$

(d) $\lim_{x\to -\infty} \dfrac{\sqrt{2x^2+1}}{x+2}$

(d) $\lim_{x\to 0} \dfrac{\sin^2(3x)}{x\tan(2x)} =$

(e) $\lim_{x\to +\infty} \dfrac{\cos(3x)}{x} =$

7. (10 pts) Sketch the graph of a function $f(x)$ satisfying all of the following conditions.

(i) The function is defined everywhere except $x = 0$ and $x = 3$;

(ii) The function is continuous everywhere except $x = 0$ and $x = 3$;

(iii) $\lim_{x\to 0} f(x) = 0$;

(iv) $\lim_{x\to 3^-} f(x) = -\infty$, $\lim_{x\to 3^+} f(x) = +\infty$;

(v) $\lim_{x\to -\infty} f(x) = 2$ and $\lim_{x\to +\infty} f(x) = 2$;

Bonus 5 pts: Find a formula for a function $f(x)$ satisfying all conditions (i)-(v) above.

8. (10 pts) (a) (3 pts) Write the general (ϵ, δ) definition for $\lim_{x\to a} f(x) = L$.

(b) (7 pts) Use the (ϵ, δ) definition to prove $\lim_{x\to -3}(10x+3) = -27$.

9. (10 pts) Choose ONE of the following:

(a) State and prove the quadratic formula.

(b) Prove the inequality $\sin x \leq x \leq \tan x$ for any $x \in [0, \pi/2)$.

(c) Use the Intermediate Value Theorem to show that the equation $x^3 = 4x - 1$ has three real solutions. Locate these solutions in intervals of length one.