Name: ____________________________ **Panther ID:** ______________

FINAL EXAM Calculus I Fall 2014

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Turn off your cell phone at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (24 pts) Compute y' (6 pts each):

(a) $y = e^x + x^\pi - \pi^e$

(b) $y = \dfrac{\sec(3x)}{2x+1}$

(c) $y = \dfrac{e^{5x} \cdot \sqrt[3]{x}}{x^2+1}$ (only logarithmic differentiation is acceptable for this one)

(d) $y = \sqrt{1 + \cos^4(5x)}$

2. (10 pts) Use implicit differentiation to find the tangent line to the curve $e^{x^2-y^2} = xy$ at $(1,1)$.

3. (24 pts) Compute the following limits, **SHOWING YOUR WORK**. If the limit does not exist or is infinite specify so.

(a) $\lim_{x\to 2} \frac{x^2-4}{|x-2|}$

(b) $\lim_{x\to -\infty} \frac{2x^5-3x^2+7x-2}{x^4+x^2+1}$

(c) $\lim_{x\to +\infty} \frac{\ln x}{\sqrt{x}}$

(d) $\lim_{x\to 0} (\cos x)^{1/x^2}$

4. (20 pts) Find each indicated antiderivative:

(a) $\displaystyle\int \left(3\cos x + \frac{2}{x} - \frac{1}{\sqrt{1-x^2}}\right) dx$

(b) $\displaystyle\int \sec^2 x \tan x \, dx$

(c) $\displaystyle\int x^2 \sqrt{x^3+4} \, dx$

5. (14 pts) Using calculus, find the dimensions of an open rectangular packaging box (with no top), made of cardboard, that satisfies all of the following requirements: (a) the volume of the box is 1000 cm^3; (b) the base is a square; (c) the amount of cardboard used to make the box should be minimal.

6. (12 pts) These are True or False questions. No partial credit. 2 points each.

a. If $\lim_{x\to 2} f(x) = f(2)$, then f is continuous at $x = 2$. **True** **False**

b. If $f'(2) = 0$, then f has a relative maximum or minimum at $x = 2$. **True** **False**

c. To compute the derivative of $\sec(\tan x)$ we must use the product rule. **True** **False**

d. If f is continuous at $x = 2$, then f is differentiable at $x = 2$. **True** **False**

e. If $\lim_{x\to 2} f(x) = f(2) = 5$, then for x sufficiently close to 2, $4.99 < f(x) < 5.01$. **True** **False**

f. If $\lim_{x\to 2} f(x) = f(2) = 5$, then for x sufficiently close to 2, $f(x) \neq 5.1$. **True** **False**

7. (12 pts) For each of the following, fill in the blanks with the most appropriate words or expressions:

(a) The definition with limit of the derivative is $f'(x) =$ ________________________.

(b) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is ________________________ on the interval (a, b).

(c) The average rate of change of a function $f(x)$ on an interval $[a, b]$, is given by ________________________.

(d) The derivative with respect to time of velocity is ________________________.

(e) A polynomial function of degree 5 can have at most ______ critical points.

(f) If $f'(5) = 0$ and $f''(5) > 0$, then x_0 is a ________________________ for the function.

8. (10 pts) (a) (6 pts) Find the local linear approximation of the function $f(x) = (1 + x)^7$ at $x_0 = 0$.

(b) (4 pts) Use part (a) to approximate 1.02^7 without using a calculator.

9. (20 pts)Given the function $f(x) = \frac{2x^2+1}{x^2-1}$ do the following:

(a) Find the domain of $f(x)$.

(b) Carefully compute $f'(x)$ and find the critical point(s). Using a sign chart for the derivative, determine the intervals over which the function is increasing and the intervals over which is decreasing.

(c) Find eventual horizontal and vertical asymptotes. Justify your answer with limits.

(d) Using the results obtained in parts (a), (b), (c), draw the graph of the function indicating the asymptotes, the coordinates and the types of the critical point(s) (the analysis of the second derivative and finding inflection points is **not** required).

10. (8 pts) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 2π mi^2/h. At what rate is the radius of the spill increasing when the radius is 2 miles?

11. (10 pts) Show, with or without Calculus, that for $x > 0$ the function $g(x) = \arctan(x) + \arctan(1/x)$, is a constant. Also, find this constant.