Quiz 1 MAC-2311 Fall 2014 **Name:** Answer Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. Compute each of the following limits. If the limit does not exist or is infinite, specify so (2.5pts each).

(a) $\lim_{x\to -1} \frac{x^2+6x+5}{x^3+x^2} \overset{\frac{0}{0}}{=}$ ← 0.5pts

$= \lim_{x\to -1} \frac{(x+1)(x+5)}{x^2(x+1)} = \lim_{x\to -1} \frac{x+5}{x^2} = \boxed{4}$

(1pt) (1.5pts)

(b) $\lim_{x\to -\infty} \frac{x^2+6x+5}{x^3+x^2} \overset{\frac{\infty}{\infty}}{=}$

$= \lim_{x\to -\infty} \frac{\cancel{x^2}\left(1+\frac{6}{x}+\frac{5}{x^2}\right)}{x^{\cancel{3}}\left(1+\frac{1}{x}\right)}$ (1.5 up to here)

$= \lim_{x\to -\infty} \frac{1+\frac{6}{x}+\frac{5}{x^2}}{x\left(1+\frac{1}{x}\right)} = \frac{1}{-\infty} = \boxed{0}$

(+1pt)

(c) $\lim_{x\to 0} \frac{x^2+6x+5}{x^3+x^2} =$

$= \lim_{x\to 0} \frac{(x+1)(x+5)}{x^2(x+1)}$ (1pt)

$= \lim_{x\to 0} \frac{(x+5)}{x^2} = \frac{5}{0^+} = \boxed{+\infty}$ (1.5pts)

important!

computing the two one-sided limits is also OK

(d) $\lim_{x\to 0} \frac{1-\cos(5x)}{x\tan(3x)} =$

(0.5pts) $= \lim_{x\to 0} \frac{(1-\cos(5x))(1+\cos(5x))}{x\tan(3x)\cdot(1+\cos(5x))} =$ $1-\cos^2(5x)$

(0.5pts) $= \lim_{x\to 0} \frac{\sin^2(5x)}{x\tan(3x)(1+\cos(5x))} =$

(1pt) $= \lim_{x\to 0} \frac{25\cancel{x}\cdot\frac{\sin^2(5x)}{(5x)^2}}{3\cancel{x}\cdot\frac{\tan(3x)}{3x}(1+\cos(5x))} = \frac{25}{3\cdot 2} = \boxed{\frac{25}{6}}$

(0.5pts)

2. (Bonus 2 pts) List all asymptotes (vertical and horizontal) for $f(x) = \frac{x^2+6x+5}{x^3+x^2}$.

Briefly justify. Note that in Pb. 1 (a), (b), (c), you computed some limits of this function.

(1pt) $x=0$ is a vertical asymptote as we showed in (c) that $\lim_{x\to 0} f(x) = +\infty$

$x=-1$ is <u>not</u> a vertical asymptote as $\lim_{x\to -1} f(x) = 4$

(1pt) $y=0$ is a horizontal asymptote for both $x\to\pm\infty$

as $\lim_{x\to\pm\infty} f(x) = 0$ (as shown in b)