Quiz 3B- MAC 2311, F'14 **NAME:** Solution Key **Panther ID:** _____

1. (10 pts) Find the derivative of each of the following functions. Simplify your answer as much as possible (2.5pts each).

(a) $y = \sin^4(2x^3) = \left(\sin(2x^3)\right)^4$

Chain Rule

$y' = 4\left(\sin(2x^3)\right)^3 \cdot \left(\sin(2x^3)\right)'$

$y' = 4\sin^3(2x^3) \cdot \cos(2x^3) \cdot (2x^3)'$

$y' = 4\sin^3(2x^3)\cos(2x^3) \cdot 6x^2$

$$\boxed{y' = 24x^2\sin^3(2x^3)\cos(2x^3)}$$

(b) $y = \ln(\sec x)$

Chain Rule

$y' = \left(\ln(\sec x)\right)' = \frac{1}{\sec x} \cdot (\sec x)'$

$y' = \frac{1}{\sec x} \cdot \sec x \cdot \tan x = \tan x$

(c) $f(x) = x^3 e^{-2x}$

Product rule

$f'(x) = (x^3)' e^{-2x} + x^3 \cdot \left(e^{-2x}\right)'$ } chain Rule

$f'(x) = 3x^2 e^{-2x} + x^3 \cdot e^{-2x} \cdot (-2)$

$$\boxed{f'(x) = x^2 e^{-2x}(3 - 2x)}$$

This exercise can be also done with logarithmic differentiation see solution of version A

(d) $g(x) = x^{5^x}$

Must use Logarithmic Differentiation

$y = x^{5^x}$ | Apply ln

$\ln y = \ln\left(x^{5^x}\right) = 5^x \cdot \ln x$

Differentiate both sides (w.r.t. x)

$(\ln y)' = (5^x \cdot \ln x)'$ ← product rule

chain Rule ↓

$\frac{1}{y} \cdot y' = 5^x \cdot \ln 5 \cdot \ln x + 5^x \cdot \frac{1}{x}$

$y' = y\left(5^x (\ln 5) \cdot \ln x + \frac{5^x}{x}\right)$

$$\boxed{y' = x^{5^x} \cdot 5^x \cdot \left((\ln 5)(\ln x) + \frac{1}{x}\right)}$$