Quiz 3A - MAC 2311, F'14 **NAME:** _____ **Panther ID:** _____

1. (10 pts) Find the derivative of each of the following functions. Simplify your answer as much as possible (2.5pts each).

(a) $y = \sin^3(3x^3) = \left(\sin(3x^3)\right)^3$

Chain Rule

$y' = 3\left(\sin(3x^3)\right)^2 \cdot \left(\sin(3x^3)\right)'$

$y' = 3\sin^2(3x^3) \cdot \cos(3x^3) \cdot (3x^3)'$

$y' = 3\sin^2(3x^3)\cos(3x^3) \cdot 9x^2$

$$\boxed{y' = 27x^2\sin^2(3x^3)\cos(3x^3)}$$

(b) $y = \ln(\sec x)$

Chain Rule

$y' = \left(\ln(\sec x)\right)' = \frac{1}{\sec x} \cdot (\sec x)'$

$y' = \frac{1}{\sec x} \cdot (\sec x)(\tan x) = \tan x$

(c) $f(x) = x^2e^{-3x}$

~~Product Rule~~

Solution with log. differentiation

$y = x^2 \cdot e^{-3x}$

$\ln y = \ln(x^2 \cdot e^{-3x}) = \ln x^2 + \ln(e^{-3x})$

$\ln y = 2\ln x - 3x$ | Apply $\frac{d}{dx}$

$(\ln y)' = 2(\ln x)' - (3x)'$

$\frac{1}{y} \cdot y' = \frac{2}{x} - 3$

$y' = y \cdot \left(\frac{2}{x} - 3\right)$

$$\boxed{y' = x^2e^{-3x}\left(\frac{2}{x} - 3\right) = xe^{-3x}(2 - 3x)}$$

(d) $g(x) = x^{2^x}$

Must use log. differentiation

$y = x^{2^x}$

$\ln y = \ln\left(x^{2^x}\right) = 2^x \cdot \ln x$

Differentiate both sides

$(\ln y)' = (2^x \cdot \ln x)'$

Chain Rule | product rule

$\frac{1}{y} \cdot y' = 2^x \cdot \ln 2 \cdot \ln x + 2^x \cdot \frac{1}{x}$

$y' = y\left(2^x \cdot \ln 2 \cdot \ln x + \frac{2^x}{x}\right)$

$$\boxed{y' = x^{2^x} \cdot 2^x\left(\ln 2 \cdot \ln x + \frac{1}{x}\right)}$$

See solution of version B for standard product rule way of solving the pb.