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Worksheet week 4 Calculus I Fall 2014

1. (3 pts) Find, if possible, a value for the constant k which will make the function $g(x)$ continuous everywhere.

$$g(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x < 0 \\ 1 + \sin x & \text{if } x \geq 0 \end{cases}.$$

We want $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = g(0)$.

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (1+\sin x) = 1+\sin 0 = 1 = g(0)$$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{1-\cos(kx)}{x^2} = \lim_{x\to 0^-} \frac{(1-\cos(kx))(1+\cos(kx))}{x^2(1+\cos(kx))} = \lim_{x\to 0^-} \frac{\sin^2(kx)}{x^2(1+\cos(kx))} =$$

$$= \lim_{x\to 0^-} \left(\frac{\sin(kx)}{x}\right)^2 \cdot \frac{1}{(1+\cos(kx))} = \frac{k^2}{2}$$

Thus, we need to have $\frac{k^2}{2} = 1$, so $\boxed{k = \pm\sqrt{2}}$

2. (4 pts) True or False. Answer and briefly justify your answer in each case.

(a) If $|f(x) + 7| \leq 3|x + 2|$ (*) for all real x, then $\lim_{x\to -2} f(x) = -7$. True

Given $\varepsilon > 0$, take $\delta = \frac{\varepsilon}{3}$. Thus, by ε-δ def.

Then if $|x+2| < \delta$ then $|f(x)+7| \underset{\text{by assumption (*)}}{\leq} 3|x+2| \underset{|x+2|<\delta}{<} 3\delta = \cancel{3}\cdot\frac{\varepsilon}{\cancel{3}}$ $\lim_{x\to -2} f(x) = -7$

(b) If $f(x)$ is continuous at $x = 2$ and $f(2) = 5$, then for x sufficiently close to 2, $f(x) > 4.95$. True.

$\lim_{x\to 2} f(x) = f(2) = 5$. Taking $\varepsilon = 0.05$ we can find $\delta > 0$ so that if $|x-2| < \delta \Rightarrow |f(x) - 5| < 0.05$

This line translates into: if $2-\delta < x < 2+\delta$ then $4.95 < f(x) < 5.05$ so statement is true

3. (4 pts) (a) Use IVT to show that the equation $x^6 + 5x^3 = 1$ has a solution in the interval $(0, 1)$.

(b) Use IVT to locate another interval of length 1 which contains a solution of the equation $x^6 + 5x^3 = 1$.

Let $f(x) = x^6 + 5x^3 - 1$. It is a polynomial so it is continuous everywhere.

(a) $\left.\begin{array}{l} f(0) = -1 < 0 \\ f(1) = 5 > 0 \end{array}\right\} \xrightarrow{\text{By IVT}}$ there is a point $c_1 \in (0,1)$ so that $f(c_1) = 0$

But $f(c_1) = 0 \Leftrightarrow c_1^6 + 5c_1^3 = 1$

so the equation has a solution in $(0,1)$

(b) $f(0) = -1 < 0$, $f(-1) = -5 < 0$, $f(-2) = 23 > 0$.

By IVT there is a point $c_2 \in (-2,-1)$ so that $f(c_2) = 0$

Thus, the equation has another real solution in $(-2,-1)$.