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Worksheet week 6 Calculus I Fall 2014

1. The curve $y = \frac{x}{1+x^2}$ is sometimes called a "serpentine" (you can check the graph on a graphing calculator or on wolframalpha.com to see why).

(a) Find the equation of the tangent line to the curve at $x = 0$.

(b) Find the coordinates of the points where the tangent line to the serpentine is horizontal.

2. Find with proof formulas for $(\cot x)'$ and $(\csc x)'$.

3. The following provides a proof for the quotient rule from the product rule.

Let $q(x) = \frac{f(x)}{g(x)}$, be the quotient of two functions $f(x)$ and $g(x)$.

The goal is to get a formula for $q'(x)$ in terms of $f'(x), g'(x), f(x), g(x)$. Proceed as follows:

Start from $q(x) \cdot g(x) = f(x)$. (Why is this true?)

Take the derivative of both sides of the above and use product rule on the left side. Then solve for $q'(x)$ and do a bit of algebra to eventually get the familiar quotient rule formula.

1.(a) Point $(0,0)$

$m_{tan} = f'(0)$

$$f'(x) = \left(\frac{x}{1+x^2}\right)' = \frac{1\cdot(1+x^2) - x\cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$f'(0) = 1$

So, by point-slope formula, tang. line is $y - 0 = 1\cdot(x-0) \iff \boxed{y = x}$

(b) We look for points x, so that $f'(x) = 0$

$f'(x) = 0 \iff 1 - x^2 = 0 \iff \boxed{x = \pm 1}$

2. $$(\cot x)' = \left(\frac{\cos x}{\sin x}\right)' \overset{\text{Q. Rule}}{=} \frac{-(\sin x)\cdot\sin x - (\cos x)\cdots}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = \underline{-\csc^2 x}$$

$$(\csc x)' = \left(\frac{1}{\sin x}\right)' = \frac{0\cdot\sin x - 1\cdot\cos x}{\sin^2 x}$$

$$= -\frac{\cos x}{(\sin x)(\sin x)} = \underline{-\cot x \cdot \csc x}$$

3. $$q(x) = \frac{f(x)}{g(x)} \Rightarrow q(x)\cdot g(x) = f(x) \Rightarrow (q(x)\cdot g(x))' = f'(x)$$

$$\Rightarrow q'(x)\cdot g(x) + q(x)\cdot g'(x) = f'(x) \Rightarrow q'(x)\cdot g(x) = f'(x) - q(x)\cdot g'(x)$$

$$\Rightarrow q'(x) = \frac{f'(x) - q(x)\cdot g'(x)}{g(x)} = \frac{f'(x) - \frac{f(x)}{g(x)} g'(x)}{g(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$