Name: Solution Key Panther ID: ____________

Worksheet week 8 Calculus I Fall 2014

1. A boat sails directly toward a 200-meters skyscraper that stands on the edge of the harbor. The "angular size" θ of the building is defined by the angle (in radians) formed by the lines from the top and the bottom of the building to the observer in the boat (picture will be drawn in class). If the boat is approaching the harbor at a constant rate of 4 meters/second, at what rate is the angular size of the building θ changing with respect to time at the moment when $\theta = \pi/6$? Give your answer in both *radians/second* and *degrees/second*. Be careful to get a correct sign for your answer and explain also why the sign that you got makes sense.

2. A plane traveling horizontally at 80 m/s over flat ground at an elevation of 3000 m releases an emergency packet. The trajectory of the packet is given by

$$x = 80t, \quad y = -4.9t^2 + 3000, \quad \text{for } t \geq 0,$$

where the origin is the point on the ground directly beneath the plane at the moment of the release, and t is the time in seconds since the moment of release.

(a) Graph the trajectory of the packet and find the coordinates of the point where the packet lands.

(b) Find dx/dt, dy/dt, explain their practical meaning and why the formulas you got for each of them makes sense.

(c) Find the angle at which the released package hits the ground.

3. The flight of a bee follows the parametric curve $x = t - \cos t$, $y = 3 - 2\sin t$, where $0 \leq t \leq 4\pi$ is the time in seconds. Use the command – plot($x = t - \cos t$, $y = 3 - 2\sin t$) – to draw this curve in wolframalpha. Be careful that the horizontal line drawn by the program is not the x-axis, but is actually the line $y = 1$.

(a) At what times is the bee flying horizontally? Find the (x, y) coordinates of the corresponding points.

(b) At what times is the bee flying vertically? Find the (x, y) coordinates of the corresponding points.

Solution for Pb. 3: Please do the graph yourself.

$$m_{tan} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\cos t}{1+\sin t}$$

(a) We want $m_{tan} = 0$, so want $\cos t = 0$, but also $1 + \sin t \neq 0$

The solutions for $0 \leq t \leq 4\pi$ are $\boxed{t = \frac{\pi}{2}}$, $\boxed{t = \frac{5\pi}{2}}$

$t = \frac{3\pi}{2}$, $t = \frac{7\pi}{2}$ are not solutions as at these points $1 + \sin t = 0$.

(b) In this case we want the slope to be undefined, so likely $t = \frac{3\pi}{2}$, $t = \frac{7\pi}{2}$ are the points we look for.

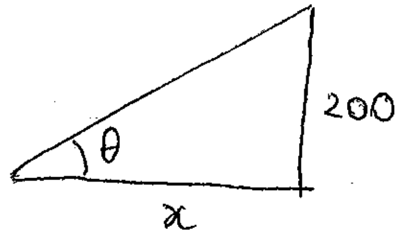
To confirm compute

$$\lim_{t \to \frac{3\pi}{2}^-} \frac{-2\cos t}{1+\sin t} = \lim_{t \to \frac{3\pi}{2}^-} \frac{-2\cos t(1-\sin t)}{(1+\sin t)(1-\sin t)} = \lim_{t \to \frac{3\pi}{2}^-} \frac{-2\cancel{\cos t}(1-\sin t)}{\cos^{\cancel{2}} t} = \frac{-4}{0^-} =$$

$$\lim_{t \to \frac{3\pi}{2}^+} \frac{-2\cos t}{1+\sin t} = \ldots = \lim_{t \to \frac{3\pi}{2}^+} \frac{-2(1-\sin t)}{\cos t} = \frac{-4}{0^+} = \boxed{-\infty}$$

Worksheet 8 - solutions

Pb. 1.



$x = x(t)$ - distance of the boat to the harbour at the moment t.

We know $\frac{dx}{dt} = -4 \, \frac{m}{s}$

We need to find $\frac{d\theta}{dt} = ?$ when $\theta = \frac{\pi}{6}$

Use $\tan\theta = \frac{200}{x}$ or, better, $\cot\theta = \frac{x}{200}$. Take $\frac{d}{dt}$ of both sides.

$$-\csc^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{200}\frac{dx}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{1}{200\csc^2\theta}\frac{dx}{dt} \quad \text{or}$$

$$\frac{d\theta}{dt} = -\frac{\sin^2\theta}{200}\frac{dx}{dt}$$

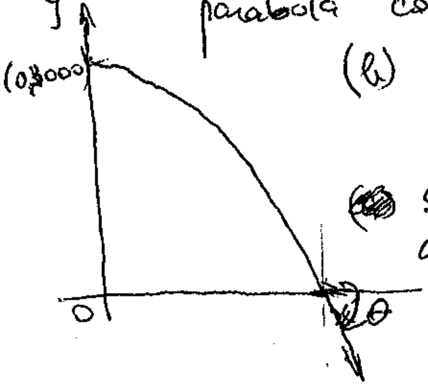
when $\theta = \frac{\pi}{6}$ $\quad \frac{d\theta}{dt} = -\frac{\sin^2\left(\frac{\pi}{6}\right)}{200}\frac{dx}{dt} = -\frac{\frac{1}{4}}{200}\cdot(-4) = \frac{1}{200} \, \frac{\text{rads}}{\text{sec}}$

or $\frac{d\theta}{dt} = \frac{1}{200}\cdot\frac{180}{\pi} \, \frac{\text{degs}}{s} = \frac{9}{10\pi} \, \frac{\text{degs}}{\text{sec}}$ $\quad \frac{d\theta}{dt} > 0$ as the angle increases.

Pb. 2 (a) Eliminating the parameter t, the trajectory is given by

$$y = -4.9\cdot\left(\frac{x}{80}\right)^2 + 3000$$

, for $x \geq 0$ so graph is a parabola concave down.



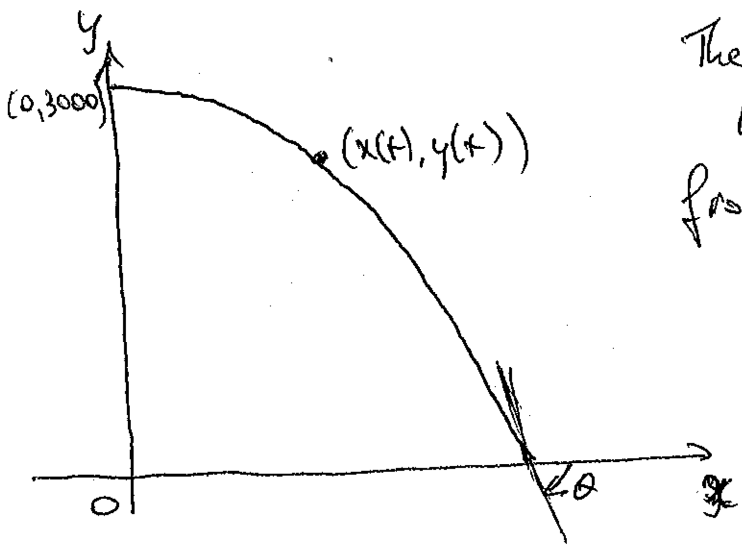
(b) $\frac{dx}{dt} = 80 \, \frac{m}{s}$ is the horizontal ~~speed~~ velocity of the object, which, assuming no air resistance, remains constant

$\frac{dy}{dt} = -9.8t$ is the vertical ~~speed~~ velocity at time t of the object. it is negative as the object is falling.

The gravitational acceleration is $9.8 \, \frac{m}{s^2}$, so the vertical speed increases linearly with t with this factor.

See next page for part (c)

Pb. 2 (c) We need to find the angle that the tangent to the curve is making with the horizontal at the x-axis intercept of the curve



The time t when the object hits the ground is obtained from solving

$y = -4.9t^2 + 3000 = 0$

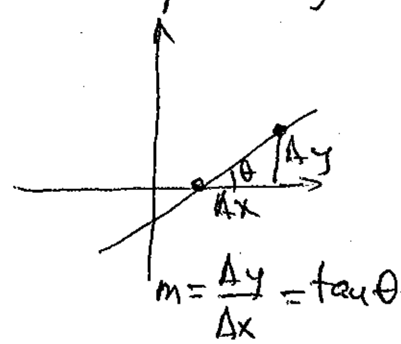
so $t = \sqrt{\frac{3000}{4.9}}\ s \approx 25s$

We find the slope of the tangent line when $t = 25s$

$$m = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=25} = \left.\frac{-9.8t}{80}\right|_{t=25} = -\frac{9.8\times 25}{80} = -3.06$$

Now for any line, the angle that the line is making with the horizontal θ is related to the slope by

$\tan\theta = m$



Thus $\tan\theta = -3.06$

so $\theta = \arctan(-3.06) \approx -72^\circ$

~~The angle is negative, because we consider~~