Worksheet 06/26 Calculus I Summer B 2018 **Group nr:** ____________

1. Consider the function $f(x) = \frac{4-x^2}{x^2-6x+8}$.

(a) Determine the points of discontinuity for $f(x)$.

(b) Use limits to understand the behavior of the function near the points of discontinuity. Are any of these removable discontinuities?

(c) Does this function have vertical asymptotes? Briefly justify your answer.

(d) Does this function have horizontal asymptotes? Justify your answer with limits.

2. Find, if possible, a value for the constant $k \geq 0$ which will make the function $g(x)$ continuous at $x = 0$.

$$g(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x < 0 \\ 1 + \sin(3x) & \text{if } x \geq 0 \end{cases},$$

3. (a) Use IVT to show that the equation $x^3 = 5x - 1$ has a solution in the interval $[2, 3]$.

(b) Use again IVT to find **three** disjoint intervals of length 1, so that in each of these intervals there is a solution for the equation $x^3 = 3x - 1$. In conclusion, you proved that the equation $x^3 = 3x - 1$ has three (distinct) real solutions.

(c) Use the bisection method to approximate one solution of the equation with an accuracy of 0.25; that is find an interval of length 1/4 which contains the solution. (A calculator may be necessary for this part.)

4. True or False questions. Answer and briefly justify your answer in each case.

(i) If $f(x)$ is a continuous function and $\lim_{x\to 3} f(x) = 4$ then $f(3) = 4$ **True** **False**

(ii) $\lim_{x\to\infty} \cos\left(\frac{\pi x^2}{2x^2+1}\right) = 0$ **True** **False**

(iii) The function $f(x) = \sec x$ is defined and is continuous for all real numbers x. **True** **False**

(iv) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$ **True** **False**

(v) If $|f(x)+7| \leq 3|x+2|$ for all real x, then $\lim_{x\to -2} f(x) = -7$ **True** **False**

(vi) If $f(x)$ is continuous at $x = 2$ and $f(2) = 5$, then for x sufficiently close to 2, $f(x) > 4.95$. **True** **False**

5. (a) Use the ϵ-δ definition of limit to prove that $\lim_{x\to 5}(20x+3) = 103$.

Challenge: (b) Use the ϵ-δ definition of limit to prove that $\lim_{x\to 5} \frac{1}{2x+3} = \frac{1}{13}$.