

Name: _____

Panther ID: _____

Exam 3

Calculus I

Fall 2015

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (18 pts) True or False? In each case, **answer and give a brief explanation of your answer.**

(a) (3 pts) If $f'(5) = 0$ and $f''(5) < 0$ then $f(x)$ has an inflection point at $x = 5$.

(b) (3 pts) If $f'(x) > 0$ for all $x \in (-\infty, +\infty)$, then $f(x)$ has no absolute maximum on $(-\infty, +\infty)$.

(c) (3 pts) For any functions f and g ,
$$\int f(x)g(x) dx = \left(\int f(x) dx \right) \left(\int g(x) dx \right) .$$

(d) (3 pts) If $f(x)$ is continuous on $[0, 4]$, then f must have an absolute minimum on this interval.

(e) (3 pts) If $f(x)$ is continuous on $[0, 4]$ and $f(0) = f(4) = 0$, then there is a point $c \in (0, 4)$ so that $f'(c) = 0$.

(f) (3 points) If $\lim_{x \rightarrow \infty} f(x) = 3$, then the graph of $f(x)$ has at least one horizontal asymptote.

2. (20 pts) Compute the following integrals:

(a) (4 pts) $\int \left(3 \cos x + \frac{1}{\sqrt{1-x^2}} \right) dx$

(b) (4 pts) $\int \frac{x^2-6}{\sqrt{x}} dx$

(c) (4 pts) $\int \tan x \sec^2 x dx$

(d) (4 pts) $\int x e^{-3x^2} dx.$

(e) (4 pts) $\int \frac{x}{x^4+1} dx$

3. (8 pts) Find the absolute maximum and absolute minimum for the function $f(x) = x^3 - 3x^2 + 1$ on the interval $[-2, 3]$. Justify your answer.

4. (16 pts) For the function $f(x) = \frac{1}{x^2 - 4x + 3}$,

(a) (4pts) Find any asymptotes. Show your reasoning both for vertical and horizontal asymptotes.

(b) (7pts) Find the intervals of increase, decrease.

(c) (5pts) Sketch the graph of f , based on the calculations in a,b. Clearly mark any asymptotes and relative extrema. You are **not required** to compute the second derivative and investigate concavity for this function.

5. (20 pts) For the function $f(x) = \ln(x^2 + 1)$,

(a) (4 pts) Find any asymptotes. Show your reasoning both for vertical and horizontal asymptotes.

(b) (5 pts) Find the intervals of increase, decrease.

(c) (5pts) Find the intervals of upward, downward concavity.

(d) (6 pts) Noting $f(0) = 0$, sketch graph based on parts a, b, c. Clearly label any asymptotes, relative extrema, and inflection points.

6. (12 pts) Compute each of the following limits

(a) (5 pts) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2}$

(b) (7 pts) $\lim_{x \rightarrow 0} (1+5x)^{\frac{2}{x}}$

7. (15 pts) Using calculus, find the dimensions of a rectangular packaging box of maximum volume that satisfies the following requirements: (a) the base (and top) of the box is a rectangle whose length is twice its width; (b) the total surface area of the box is 1500 cm^2 .

8. (12 pts) (a) (6 pts) Suppose an object is moving on a straight line with constant acceleration a . Assume that the initial velocity is v_0 and the initial position is s_0 . Use integration to find the formulas for the velocity $v(t)$ and the position $s(t)$ of the object at time t .

(b) (6 pts) An arrow is shot vertically from ground level. After 3 seconds the arrow is 120 ft above the ground. What is the initial velocity of the arrow? You can assume that there is no friction and that the acceleration is the gravitational acceleration $a = -32 \text{ ft/s}^2$.