Name: Solution Key　　　　Panther ID: ____________

**Homework 2 - Due Thursday, Sept. 17**　　Calculus I　　Fall 2015

**1.** (3 pts) Find, if possible, a value for the constant $k$ which will make the function $g(x)$ continuous everywhere.

$$g(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x < 0 \\ 1+\cos x & \text{if } x \geq 0 \end{cases},$$

We want: $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = g(0)$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{1-\cos(kx)}{x^2} = \lim_{x\to 0^-} \frac{(1-\cos(kx))(1+\cos(kx))}{x^2(1+\cos(kx))} = \lim_{x\to 0^-} \frac{\sin^2(kx)}{x^2(1+\cos(kx))} =$$

$$= \lim_{x\to 0^-} \frac{k^2\sin^2(kx)}{(kx)^2(1+\cos(kx))} = \frac{k^2}{2}$$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (1+\cos x) = 1+\cos 0 = 2 = g(0)$$

Thus, we want $\frac{k^2}{2} = 2$, so $k^2 = 4$ so $\boxed{k = \pm 2}$ will make $g(x)$ continuous everywhere.

**2.** (4 pts) True or False. Answer and briefly justify your answer in each case.

(a) If $|f(x)+5| \leq 7|x+3|$ for all real $x$, then $\lim_{x\to -3} f(x) = -5$.

$\boxed{\text{True}}$: Given $\varepsilon > 0$, choose $\delta = \frac{\varepsilon}{7}$

Then if $|x+3| < \delta \Rightarrow |f(x)+5| \leq 7|x+3| < 7\delta = \cancel{7}\cdot\frac{\varepsilon}{\cancel{7}}$ so indeed $\lim_{x\to -3} f(x) = -5$.

(b) If $f(x)$ is continuous at $x = 2$ and $f(2) = 5$, then for $x$ sufficiently close to 2, $f(x) < 5.002$.

$\boxed{\text{True}}$: $f(x)$ cont. at $x=2$ and $f(2)=5 \Rightarrow \lim_{x\to 2} f(x) = f(2) = 5$

So given $\varepsilon = 0.002$, there is $\delta > 0$ so that if $|x-2| < \delta$ then $4.998 < f(x) < 5.002$

**3.** (4 pts) (a) Use IVT to show that the equation $x^3 = 3x - 1$ has a solution in the interval $[0, 1]$.

(b) Use IVT to show that the equation $x^3 = 3x - 1$ has three real solutions and find intervals of length 1 containing each solution.

(a) $x^3 = 3x-1 \Leftrightarrow x^3 - 3x + 1 = 0$.

Let $f(x) = x^3 - 3x + 1$ continuous everywhere since it is a polynomial.

$\left.\begin{array}{l} f(0) = 1 > 0 \\ f(1) = -1 < 0 \end{array}\right\} \xrightarrow{\text{By I.V.T.}}$ there is a value $x_1 \in [0,1]$ so that $f(x_1) = 0$.

(b) Evaluate $f$ at other integer inputs:

$f(-1) = 3 > 0$, $f(-2) = -1 < 0 \xrightarrow{\text{I.V.T.}}$ there is $x_2 \in [-2,-1]$ s.t. $f(x_2) = 0$.

$f(1) = -1 < 0$, $f(2) = 3 > 0 \xrightarrow[\text{I.V.T.}]{}$ there is $x_3 \in [1,2]$ s.t. $f(x_3) = 0$.

Thus the equation has 3 real roots, one in each of the intervals

$[-2,-1]$, $[0,1]$, $[1,2]$.