NAME: Solution Key **Panther ID:** _____

Quiz 1 MAC-2311 Fall 2015

To receive full credit you should completely justify each answer.

1. (10 pts) Compute each limit. If the limit does not exist or is infinite specify so (2.5pts each).

(a) $\lim_{x\to 3} \dfrac{x^2-5x+6}{3x-x^2} \overset{\frac{0}{0}}{=}$

$= \lim_{x\to 3} \dfrac{(x-3)(x-2)}{x(3-x)} =$

$= \lim_{x\to 3} \dfrac{(x-3)(x-2)}{-x(x-3)} = \boxed{-\dfrac{1}{3}}$

(b) $\lim_{x\to 0^-} \dfrac{x^2-5x+6}{3x-x^2} = \dfrac{6}{0^-} = \boxed{-\infty}$

As $x\to 0^-$, $3x$ is a small negative number

$-x^2$ is also a small negative number

ok also

$\lim_{x\to 0^-} \dfrac{(x-3)(x-2)}{-x(x-3)} = \lim_{x\to 0^-} \dfrac{x-2}{-x} = \dfrac{-2}{0^+} = -\infty$

(c) $\lim_{x\to+\infty} \dfrac{x^2-5x+6}{3x-x^2} =$

$= \lim_{x\to+\infty} \dfrac{x^2\left(1-\frac{5}{x}+\frac{6}{x^2}\right)}{x^2\left(-1+\frac{3}{x}\right)} = \boxed{-1}$

Solution with Garbage rule

$\lim_{x\to+\infty} \dfrac{x^2-5x+6}{3x-x^2} = \lim_{x\to+\infty} \dfrac{x^2}{-x^2} = \boxed{-1}$

receives 2pts out of 2.5 pts.

(d) $\lim_{x\to-\infty} \dfrac{\sqrt{9x^2+1}}{x+6} =$

$= \lim_{x\to-\infty} \dfrac{\sqrt{x^2\left(9+\frac{1}{x^2}\right)}}{x\left(1+\frac{6}{x}\right)} = \lim_{x\to-\infty} \dfrac{|x|\sqrt{9+\frac{1}{x^2}}}{x\left(1+\frac{6}{x}\right)}$

$= \lim_{x\to-\infty} \dfrac{-x\cdot\sqrt{9+\frac{1}{x^2}}}{x\cdot\left(1+\frac{6}{x}\right)} = -\dfrac{\sqrt{9}}{1} = \boxed{-3}$

2. (Bonus 2 pts) List all asymptotes (vertical and horizontal) of $f(x) = \dfrac{x^2-5x+6}{3x-x^2}$.

Briefly justify. Note that in Pb. 1 (a), (b), (c), you computed some limits of this function.

From (a), $x=3$ is not a vertical asymptote for $f(x)$

At $(3, -\frac{1}{3})$ there will be a circle on the graph of $f(x)$ (point is not included)

$x=0$ is a vertical asymptote (as shown in (b))

From (c) $y=-1$ is a horizontal asymptote when $x\to+\infty$

since $\lim_{x\to-\infty} \dfrac{x^2-5x+6}{3x^2-x^2} = -1$, $y=-1$ is also a horiz. asymptote when $x\to-\infty$.