

To receive credit you MUST SHOW ALL YOUR WORK.

1. (3 pts) Solve the initial value problem:

$$\frac{dy}{dx} = 5e^x - 3\sqrt{x}, \quad y(0) = 0$$

$$y(x) = \int (5e^x - 3\sqrt{x}) dx = \int (5e^x - 3x^{\frac{1}{2}}) dx = 5e^x - 2 \cdot \frac{2}{3} x^{\frac{3}{2}} + c$$

$$y(0) = 0 \Rightarrow 0 = 5e^0 - 2 \cdot 0^{\frac{3}{2}} + c \Rightarrow 0 = 5 + c \Rightarrow c = -5$$

2. (9 pts) Evaluate each integral:

Thus $y(x) = 5e^x - 2x^{\frac{3}{2}} - 5$

$$(a) \int \left(\frac{1}{3x} - \frac{2}{\sqrt{1-x^2}} + \pi^2 \right) dx = \int \left(\frac{1}{3} \cdot \frac{1}{x} - \frac{2}{\sqrt{1-x^2}} + \pi^2 \right) dx$$

$$= \frac{1}{3} \ln|x| - 2 \arcsin(x) + \pi^2 \cdot x + c$$

$$(b) \int \frac{\sec^2(2/x)}{x^2} dx =$$

sub: $u = \frac{2}{x}$

$$du = -\frac{2}{x^2} dx$$

$$-\frac{1}{2} du = \frac{1}{x^2} dx$$

$$= \int \sec^2(u) \cdot \left(-\frac{1}{2} du\right) = -\frac{1}{2} \int \sec^2(u) du$$

$$= -\frac{1}{2} \tan(u) + c = -\frac{1}{2} \tan\left(\frac{2}{x}\right) + c$$

$$(c) \int (1+3\cos x)^5 \sin x dx =$$

$$u = 1 + 3\cos x$$

$$du = -3\sin x dx$$

$$-\frac{1}{3} du = \sin x dx$$

$$= \int u^5 \cdot \left(-\frac{1}{3}\right) du$$

$$= -\frac{1}{3} \cdot \frac{u^6}{6} + c$$

$$= -\frac{1}{18} (1+3\cos x)^6 + c$$