NAME: ______________________________ **Panther ID:** ____________

Worksheet Sep. 3 - MAC 2311, Fall 2015

1. In the following picture (to be inserted) B is the point of coordinates $(1, 0)$, A is a point on the unit circle in the first quadrant, and denote by θ the angle $\angle AOB$. Denote also by C the point at the intersection of the line OA with the vertical line through B.

(a) Assuming $\theta = 20^\circ$ compute each of the following: the x and y coordinates of the point A; the length of the segment BC; the length of the arc AB; the area of the triangle AOB; the area of the sector AOB; the area of the triangle OBC. It's OK if your answers contain trigonometric expressions. Do not try to evaluate them.

(b) With the same picture as above, for an arbitrary value of the angle θ, (θ in **radians** between 0 and $\pi/2$), find expressions in terms of θ for the area of the triangle AOB, the area of the sector AOB, the area of the triangle OBC.

(c) Considering the obvious inequality between the areas, what (double) inequality have you proved?

(d) After a couple of algebraic manipulations, that you should describe, your inequality in part (c) can be re-written as

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

(e) Explain intuitively in one or two sentences why the (double) inequality in (d) implies that

$$\lim_{\theta\to 0^+} \frac{\sin\theta}{\theta} = 1 \ .$$

There is a rigorous reason for this last step, the so called "Squeeze Theorem" for limits, which will be briefly explained in lecture. With steps (a)-(e), you discovered the proof of the famous limit

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ .$$

This limit has many computational applications, some of which you will discover in the next exercise.

2. Compute each of the following limits:

(*a*) $\lim_{x\to 0} \dfrac{\sin(5x)}{x} =$ $\qquad$ $\lim_{x\to 0} \dfrac{\sin(ax)}{x} =$

(*b*) $\lim_{x\to +\infty} \dfrac{\sin x}{x}$

(*c*) $\lim_{x\to 0} \dfrac{\tan(3x)}{x} =$ $\qquad$ $\lim_{x\to 0} \dfrac{\tan(bx)}{x} =$

(*d*) $\lim_{x\to 0} \dfrac{1-\cos(x)}{x}$

(*e*) $\lim_{x\to 0} \dfrac{\tan^2(3x)}{x\sin(5x)}$

(*f*) $\lim_{x\to +\infty} x\tan(3/x)$ Hint: Use substitution technique.