NAME: ________________________________ **Panther ID:** ____________

Worksheet on the limit definition - MAC 2311, Fall 2015

1. Find the distance between:

(a) 2 and 5; (b) 5 and 2; (c) -3 and 2;

(d) 5.2 and -3.7; (e) a and b, with a and b real numbers

2. (a) Sketch the set of points $(x, 0)$ on the x-axis such that $|x - 3| < 0.7$. Then sketch the set of points $(x, 0)$ on the x-axis such that $0 < |x - 3| < 0.7$. How are these two sets different?

(b) Describe the set of points on the x axis whose distance from $(5, 0)$ is less than 0.1 in two ways:

(i) Sketch this set;

(ii) Write an inequality characterizing this set (hint: look at (2a)).

3. Describe the set of points on the y axis whose distance from $(0, 3)$ is less than 0.2 in two ways:

(i) Sketch this set;

(ii) Write an inequality characterizing this set (hint: look at (2a)).

4. In this problem, we will use the ϵ-δ definition to prove that $\lim_{x\to 5}(2x+3) = 13$.

(a) Identify $f(x)$, a, L in this case.

(b) Compute $|f(x) - L|$ in this case (you want in your result to see the expression $|x - a|$).

(c) Using your computation in (b), show that if $|x - 5| < 0.1$ then $|f(x) - 13| < 0.2$.

More generally, show that if $\delta > 0$ is a positive number (not yet specified) then if $|x - 5| < \delta$ then it follows, in this case, that $|f(x) - 13| < 2\delta$.

(d) Based on part (c), if $\epsilon > 0$ is given, how would you choose $\delta > 0$ in this case?

(e) With your choice from part (d), show that if $|x - 5| < \delta$ then $|f(x) - 13| < \epsilon$.

5. Repeat the steps in the previous problem to show that $\lim_{x\to 3}(100x-1) = 299$.

6. Prove that $\lim_{x \to -3} (2x-7) = -13$.

7. True or false. Answer and justify your answer.

(a) If $\lim_{x\to 2} f(x) = f(2) = 5$, then $4.9 < f(x) < 5.1$ for all x in a small enough interval around 2.

(b) If $\lim_{x\to 2} f(x) = f(2) = 5$, then $f(x) \neq 4.99$ for all x in a small enough interval around 2.

7. (Challenge problem) Prove $\lim_{x\to 2} x^2 = 4$.