

Panther ID: _____

NAME: _____

MAC 2311 - Fall 2015 – Worksheet 0915 (Continuity)

1) Use the checklist for continuity to show that the function

$$f(x) = (x + 2x^3)^4$$

is continuous at $x = -1$.

2) Use the checklist for continuity to show that the function

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$

is discontinuous at $x = 1$ in the following steps.

a) Is $f(x)$ defined at $x = 1$? If so, what is $f(1)$?

b) Compute $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$.

c) Does $\lim_{x \rightarrow 1} f(x)$ exist? Explain why or why not.

d) Is $\lim_{x \rightarrow 1} f(x)$ equal to $f(1)$?

e) Which of the three points in the checklist does $f(x)$ not satisfy?

3) Define

$$h(t) = \frac{t^2 + 3t - 10}{t - 2}$$

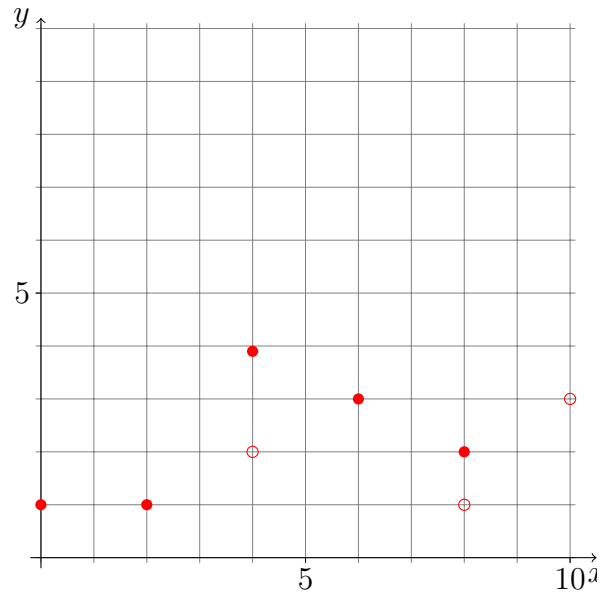
a) Use the checklist for continuity to explain why $h(t)$ is not continuous at $t = 2$.

b) Define a new function $H(t)$ by

$$H(t) = \begin{cases} h(t) & \text{if } t \neq 2 \\ a & \text{if } t = 2 \end{cases}$$

where a is some unknown number. Why is $\lim_{x \rightarrow 2} H(t) = \lim_{x \rightarrow 2} h(t)$? What value for a would make $H(t)$ continuous at $t = 2$?

4) Let $f(x)$ be a function whose graph is given by the following:



Find the points x between 0 and 10 where $f(x)$ is not continuous. At each of these points, explain which items of the checklist $f(x)$ fails to satisfy.

5) For the following functions, identify the domain of each and state where each is continuous. Also, explain why each function is discontinuous at each point of discontinuity.

a) $f(t) = \frac{t^2 - 3t + 2}{t^2 - 4}$

b) $g(x) = \frac{t^2 - 3t + 2}{t^2 + 4}$

c) $h(s) = \sqrt{s^2 - 6}$

d) $k(x) = |x - 1|$

6) (a) The definition of the function $f(x) = \begin{cases} 4x^3 - 3x^2 & \text{for } x \leq 2 \\ 3x + a & \text{for } x > 2 \end{cases}$

contains the constant a . Find the value(s) of a which make $f(x)$ continuous everywhere.

(b) The definition of the function $g(x) = \begin{cases} 1 + \cos x & \text{for } x \leq 0 \\ \frac{\tan(kx)}{x} & \text{for } x \geq 0 \end{cases}$

contains the constant k . Find the value(s) of k which make $g(x)$ continuous at 0.

With the value of k that you found, is $g(x)$ continuous everywhere? Explain your answer.

7) (a) Use the intermediate value theorem to show that the equation $x^4 = 5x^3 - 1$ has a solution in the interval $[0, 1]$.

(b) Approximate the solution in part (a) with an accuracy of 0.25; that is find an interval of length $1/4$ which contains the solution.

(c) Use IVT one more time to show that the equation $x^4 = 5x^3 - 1$ has at least a second real solution and find an interval of length 1 containing this second solution

8) For the following functions, identify the domain of each and state where each is continuous. Also, explain why each function is discontinuous at each point of discontinuity.

1. $f(x) = \cos(3x)$

2. $g(x) = \sec(3x)$