Worksheet 09-24 – on Sections 2.3 and 2.2:

LECTURE INTRO: Review limit definition of derivative, notation, and formulas for derivatives of c, x, x^2, $\sqrt{x}$.

1. Use the limit definition of the derivative, to find the formula for the derivative of $\frac{1}{x}$.

2. In this problem, you will derive an expression for the derivative of x^n where $n = 1, 2, 3, \ldots$.

a) Show the following equalities:

$$\begin{aligned} a^2 - b^2 &= (a-b)(a+b), \\ a^3 - b^3 &= (a-b)(a^2 + ab + b^2), \\ a^4 - b^4 &= (a-b)(a^3 + a^2b + ab^2 + b^3). \end{aligned}$$

b) Using the equalities in (a) as examples, find a polynomial $p(a,b)$ so that

$$a^n - b^n = (a-b)p(a,b).$$

c) Write down the limit definition of $(d/dx)x^n$ and use the result from (b) to compute this limit.

3. Compute the following derivatives:

a) $\frac{d}{dx}\pi^2$

b) $\frac{d}{dx}x^{12}$

LECTURE BREAK: Derive rules for $(cf)'$ and $(f+g)'$.

4. Use this table of values

x	**1**	**2**	**3**	**4**	**5**
$f'(x)$	3	5	2	1	4
$g'(x)$	2	4	3	1	5

to calculate the value of these derivatives:

1. $\displaystyle \frac{d}{dx}\left[\frac{3}{2}f(x)\right]\Bigg|_{x=1}$

2. $\displaystyle \frac{d}{dx}\left[f(x) - g(x)\right]\Bigg|_{x=2}$

3. $\displaystyle \frac{d}{dx}\left[3g(x) - 4f(x)\right]\Bigg|_{x=3}$

5. Compute the following derivatives:

a) $\frac{d}{dx}\left(4x^3\right)$

b) $\frac{d}{dx}\left(7x^5 - 2x^4 + 3x + 12\right)$

c) A general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ \dots\ + a_2 x^2 + a_1 x + a_0\ ,$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are constant coefficients, $a_n \neq 0$. Find the derivative of P.

6. Find the equation of the line tangent to the curve $y = 4x^4 - 3x^3$ at $(2, 40)$

7. The equation $y'' + y' - 2y = x^2$ is called a *differential equation* because it involves an unknown function $y(x)$ and its derivatives y' and y''. Find constants A, B, C such that the function $y = Ax^2 + Bx + C$ satisfies this equation. (There is a separate course dedicated to Differential Equations.)

8. Pb. 73 section 2.3 textbook

LECTURE BREAK: Prove that differentiability implies continuity and note the converse is not true.

9. Consider the function

$$g(x) = \begin{cases} x^3 + 1 & \text{if } x \leq 1 \\ 3 - x & \text{if } x > 1 \end{cases}$$

(a) Is $g(x)$ continuous everywhere? Justify your answer.

(b) Is $g(x)$ differentiable everywhere? Justify your answer.

(c) Find a formula for $g'(x)$ at the points where it exists.

10. Pb. 26 section 2.2 textbook

11. Pb. 42 section 2.2 textbook