MAC 2311: Worksheet 10/01/2015 – Chain rule:

LECTURE INTRO: Give or derive

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

1) We will use the chain rule to compute the derivative of $(\sqrt{1+x^2})$.

a) Find functions $f(u)$ and $g(x)$ so that $\sqrt{1+x^2} = f(g(x))$.

b) Compute $f'(u)$ and $g'(x)$.

c) Use the chain rule formula and your computations in (b) to compute

$$\frac{d}{dx}\sqrt{1+x^2}.$$

2) We will use the chain rule to compute the derivative of $\tan(t^2)$.

a) Find functions $f(u)$ and $g(t)$ so that $\tan(t^2) = f(g(t))$.

b) Compute $f'(u)$ and $g'(t)$.

c) Use the chain rule formula and your computations in (b) to compute

$$\frac{d}{dt}\tan\left(t^2\right).$$

3) Use the chain rule and other rules of differentiation as needed to compute the derivatives of the following functions

1. $f(x) = \dfrac{1}{\sqrt{1+x^2}}$
2. $v(t) = \cos^2(3t)$
3. $h(x) = x\left(x^9+2\right)^{1/2}$

4) Consider the function $y = \sqrt{x^2-9}$. Find the equation of the line tangent to this function at $x = 5$.

5) Suppose that the energy used by a factory is given (in megawatt-hours, MWh) by

$$E(t) \quad = \quad 200t + \frac{1200}{\pi} \sin\left(\frac{\pi t}{12}\right)$$

where $0 \leq t \leq 24$ is measured in hours after noon.

1. Calculate the power, $P(t) = E'(t)$, consumed by this factory. What are the units in this case?
2. When is the power consumption highest?
3. When is the power consumption lowest?
4. Make a sketch of the power consumption from $t = 0$ to $t = 24$.

6) Use the chain rule and other rules of differentiation as needed to compute the derivatives of the following functions

1. $g(x) = \sin(x\cos(x))$
2. $f(x) = \sqrt{\csc\left(\sin^2 x\right)}$
3. $j(x) = \sec(3 + x^2\tan(3x))$