NAME: ______________________________

MAC 2311: Worksheet #9 **10/06/2015**

Derivatives of Exponential and Logarithmic Functions

LECTURE: Review of logarithms and exponentials.

1) Without using a calculator, compute the following:
   a) $\log_2(8)$
   b) $\log_5(\frac{1}{25})$
   c) $\log_{1/3}(9)$

2) If $\log_b(A) = 5$, $\log_b(B) = 3$, and $\log_b(C) = 2$, compute

$$\log_b\left(\frac{A^2}{B^4C^3}\right).$$

3) Solve the equation $\log_2(x^2 + 1) = 1$.

4) Solve the equation $5^{3x} = 7$.

LECTURE: the number e as a limit, e^x, $\ln(x)$, and $(e^x)'$.

5) Find each of the following derivatives. What are you using in each case?

(a) $\frac{d}{dx}(e^{7x}) =$

(b) $\frac{d}{dx}(e^{x^3}) =$

(c) $\frac{d}{dx}(e^{x^2-10x}) =$

(d) $\frac{d}{dx}(e^{f(x)}) =$

(e) $\frac{d}{dx}(e^{\pi}) =$

6) Use the trick that $2^x = e^{\ln(2^x)} = e^{x(\ln 2)}$, to find a formula for

$$\frac{d}{dx}(2^x)$$

In general, if a is a positive constant, $\frac{d}{dx}(a^x) = (\ln a)a^x$.

7) Compute the derivative of each of the following functions:

(a) $y = \sin(3^x)$ (b) $y = (\sin x)3^x$ (c) $y = 3^{\sin x}$

8) Find the values of x, if any, where $f(x) = x^2 e^{-3x}$ has a horizontal tangent line.

LECTURE BREAK: Use $e^{\ln x} = x$ to find $(\ln x)' = \frac{1}{x}$. Show also $(\log_a x)' = \frac{1}{(\ln a)x}$.

9) Compute the derivative of each of the following functions:

(a) $y = \ln(x^2)$ (b) $y = (\ln x)^2$

(c) $y = \log(\log x)$ (d) $y = x^2 \log_2 x$

(e) $y = \ln(\sec x)$

(f) $y = \ln(\sec x + \tan x)$

(g) $y = \ln\left(\frac{\sqrt{1-3x^2}}{\cos^2 x}\right)$

10) Explain why the y-intercept of any tangent line to the curve $y = \ln x$ must be one unit less than the y-coordinate of the point of tangency. *Hint:* Write the equation of the tangent line to $y = \ln x$ at an arbitrary point $x_0 = a$.

11) Many population growth/decay models follow an exponential model. An exponential model is characterized by the property that the rate of change of the population is proportional to its size. Let $P(t) = P_0 e^{kt}$ be a certain population at time t, where P_0 and k are constant parameters.

(a) What is the meaning of P_0?

(b) Show that $P(t) = P_0 e^{kt}$ satisfies $P'(t) = kP(t)$, so, indeed, the rate of change of the population is proportional to its size, k being the constant of proportionality. (The constant k is also called the *relative growth rate*.)

12) A bacteria culture initially contains 100 cells and grows exponentially. After one hour the population has increased to 420.

(a) Find an expression for the number of bacteria after t hours.

(b) Find the number of bacteria after 3 hours.

(c) Find the rate of growth after 3 hours.

(d) When will the population reach 10,000?

Practice: (Don't turn these in.)
§3.2 # 1-27 odd, 31, 35-41odd, 45*, 47*
§3.3 # 15-41 odd, 71-74, 77, 79