**NAME:** ______________________________

**MAC 2311: Worksheet #10** **10/08/2015**

**Derivatives of Inverse Trigonometric Functions. Logarithmic Differentiation.**

LECTURE: Definition of $\arcsin(x)$, $\arctan(x)$. Derivative of $\arcsin(x)$.

1) Without using a calculator, compute:

a) $\arcsin(1/2)$

b) $\arctan(1)$

c) $\sin(\arcsin(1/5))$

d) $\arcsin(\sin(\pi/5))$

e) $\arctan(\tan(-3\pi/4))$

f) $\arcsin(\sin(3\pi/4))$

2) Compute the following derivatives:

a) $\frac{d}{dx}(x^3 \arcsin(3x))$

b) $\frac{d}{dx}\left(\frac{\sqrt{x}}{\arcsin(x)}\right)$

c) $\frac{d}{dx}[\ln(\arcsin(e^x))]$

d) $\frac{d}{dx}[\arcsin(\cos x)]$

The result of part d) might be surprising, but there is a reason for it. If you find it, it will also lead you to a simple proof for the derivative of $\arccos x$!

3) In this problem, you will compute $\frac{d}{dx}\arctan(x)$

a) Using the chain rule, differentiate both sides of the equality $\tan(\arctan(x)) = x$ and solve the resulting equation for $\frac{d}{dx}\arctan(x)$.

b) Let $\theta = \arctan(x)$ so $\tan(\theta) = x$. Draw a right triangle with vertices $A$, $B$, and $C$ and angles $\angle ABC = \pi/2$ and $\angle BAC = \theta$. If the length of the side $AB$ is $|AB| = 1$, find the lengths $|BC|$ and $|AC|$ in terms of $x$.

c) Using the triangle you drew in (b), find $\sec(\arctan(x))$.

d) Combine your answers for (c) and (a) to get $\frac{d}{dx}\arctan(x)$.

4) Compute the following derivatives:

a) $\frac{d}{dx}[\arctan(e^x)]$

b) $\frac{d}{dx}[e^x \arctan(x)]$

c) $\frac{d}{dx}[\sin(\arctan(x))]$

d) $\frac{d}{dx}[\arctan(\arcsin(x^2))]$

LECTURE BREAK: Logarithmic differentiation. Show the example $(x^x)'$

5) Use logarithmic differentiation to find the derivative of each of the following functions:

(a) $y = x^{\sin x}$

(b) $y = \frac{x^2 \sqrt[3]{5+x^2}}{(x+2)^5}$

6) (a) We proved the power rule $(x^n)' = nx^{n-1}$ for the case when $n$ was a positive integer and in some other special cases. Now use logarithmic differentiation to show that the power rule $(x^r)' = rx^{r-1}$ holds for *any* real constant $r$.

(b) Use logarithmic differentiation to prove the product rule.

(c) Use logarithmic differentiation to prove the quotient rule.

LECTURE BREAK: Implicit differentiation; Show one or two examples.

7) For each of the following implicitly defined functions, find $\dfrac{dy}{dx}$:

a) $y^4 - 3y^3 - x = 3$

b) $\cos(xy) = x - y$

8) Consider the function implicitly defined by $y^4 = x + y$.

a) Find an expression for the derivative $\dfrac{dy}{dx}$.

b) Find the equation of the line tangent to this function at the point (0,1).

c) Find where the tangent line is vertical.

**Practice:** (Don't turn these in.) 3.3 # 43-53 odd, 65 – Inverse trig differentiation problems.
3.1 # 1-13odd, 19, 25, 27, 29*, 33* – Implicit diff problems.
Logarithmic Differentiation problems were recorded on the previous worksheet (in 3.2).