NAME: _____

MAC 2311: Worksheet #11   **10/13/2015**

Implicit differentiation. Related rates.

LECTURE: Functions defined implicitly, implicit differentiation.

1) For each of the following implicitly defined functions, find $\dfrac{dy}{dx}$:

    a) $y^4 - 3y^3 - x = 3$ b) $\cos(xy) = x - y$

2) Consider the function implicitly defined by $y^4 = x + y$.

    a) Find an expression for the derivative $\dfrac{dy}{dx}$.

    b) Find the equation of the line tangent to this function at the point (0,1).

    c) Find where the tangent line is vertical.

LECTURE: Related rates.

3) (Lysis of a bacterium): a spherical bacterium has its cell wall perforated. As a result, water flows into the bacterium at 100 cubic micrometers per second.

    a) What is the rate of change of the radius at the instant that the radius is 30 micrometers

    b) What is the rate of change of the surface area at the instant that the radius is 30 micrometers?

4) Suppose that a cylindrical tank is being filled with water at a rate of 100 cm^3/hr. If the tank has a radius of 50 cm, how is the height of water in the tank changing when the tank is 100 cm full?

5) A ten foot ladder is leaning against a wall when it starts to slip. Suppose the base of the ladder is moving away from the wall at a rate of 3 feet per second. What is the rate at which the top of the ladder is descending when the top is five feet above the ground?

6) A boat is pulled towards a dock by a rope that runs through a ring on the dock mounted 6 ft above the bow of the boat. The rope is drawn in at a rate of 2 ft/sec.

    a) How fast is the boat approaching the dock when 10 ft of rope is still out?

    b) At what rate is the angle between the rope and the vertical changing at that time?

7) (Ships passing in the night:) the Carnivore Princess is sailing East out of Miami at a speed at 30 km/hr, while the Royal Flounder is sailing North towards Miami at speed 25 km/hr. At some instant, the Princess is 30 km East of Miami, while the Flounder is 40 km South of Miami. At that instant, are the ships getting closer or getting farther apart? At what rate?

8) A telescope on the ground is tracking a rocket which is rising vertically from a launchpad. The telescope is 5 kilometers from the launchpad and denote by θ the angle with respect to which the telescope observes the rocket above the ground. Suppose that at the moment when the rocket is 10km above the ground, the angle θ is increasing at a rate of one degree per second. What is the vertical speed of the rocket at that moment?

9) Suppose f is a differentiable function which is also one to one and let f^{-1} denote the inverse of f. Obtain a formula relating the derivative of f^{-1} to the derivative of f. *Hint:* One way to do this is to use implicit differentiation: if $y = f^{-1}(x)$, then $x = f(y)$, and take d/dx of both sides.

Practice: (Don't turn these in.)

§3.1 (Implicit differentiation) # 1-13odd, 19, 25, 27, 29, 33.
§3.4 (Related rates) # 5,7,8,12,13,17-20all,24,29,32,45.