**Worksheet 10-15-15** - MAC 2311, Fall 2015 **NAME:** ____________________

LECTURE: Local linear approximation formula

**1.** Use local linear approximation to estimate the value of $\sqrt[3]{1003}$. Be sure to specify the function $f(x)$ and the point $x_0$ that you will choose for the approximation.

**2.** (a) Find the local linear approximation of $f(x) = \cot x$ at $x_0 = \pi/4$.

(b) Use the local linear approximation in part (a) to estimate $\cot(43°)$.

LECTURE BREAK: Differentials. Error and percentage error.

**3.** Let $y = \frac{1}{x}$. Find $\Delta y$ and $dy$ at $x = 1$ if $dx = \Delta x = 0.5$. On a graph of $y = \frac{1}{x}$, mark the quantities $\Delta y$ and $dy$ which you computed.

**4.** The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm.

(a) Use differentials to estimate the error in the calculated volume.

(b) Use differentials to estimate the percentage error in the calculated volume.

**5.** Use differentials to estimate the volume of paint needed to apply a coat of paint 0.1 cm thick to a hemispherical dome with diameter 50 m.

**Practice:** (Don't turn these in.)

§3.5 (Local linear approximation) # 1-9odd, 23, 27, 29, 34, 51, 55, 63, 67.

**The Related Rates problems from previous worksheet.**

3) (Lysis of a bacterium): a spherical bacterium has its cell wall perforated. As a result, water flows into the bacterium at 100 cubic micrometers per second.

a) What is the rate of change of the radius at the instant that the radius is 30 micrometers

b) What is the rate of change of the surface area at the instant that the radius is 30 micrometers?

4) Suppose that a cylindrical tank is being filled with water at a rate of 100 $\text{cm}^3/\text{hr}$. If the tank has a radius of 50 cm, how is the height of water in the tank changing when the tank is 100 cm full?

5) A ten foot ladder is leaning against a wall when it starts to slip. Suppose the base of the ladder is moving away from the wall at a rate of 3 feet per second. What is the rate at which the top of the ladder is descending when the top is five feet above the ground?

6) A boat is pulled towards a dock by a rope that runs through a ring on the dock mounted 6 ft above the bow of the boat. The rope is drawn in at a rate of 2 ft/sec.

a) How fast is the boat approaching the dock when 10 ft of rope is still out?

b) At what rate is the angle between the rope and the vertical changing at that time?

7) (Ships passing in the night:) the Carnivore Princess is sailing East out of Miami at a speed at 30 km/hr, while the Royal Flounder is sailing North towards Miami at speed 25 km/hr. At some instant, the Princess is 30 km East of Miami, while the Flounder is 40 km South of Miami. At that instant, are the ships getting closer or getting farther apart? At what rate?

8) A telescope on the ground is tracking a rocket which is rising vertically from a launchpad. The telescope is 5 kilometers from the launchpad and denote by $\theta$ the angle with respect to which the telescope observes the rocket above the ground. Suppose that at the moment when the rocket is 10km above the ground, the angle $\theta$ is increasing at a rate of one degree per second. What is the vertical speed of the rocket at that moment?

9) Suppose $f$ is a differentiable function which is also one to one and let $f^{-1}$ denote the inverse of $f$. Obtain a formula relating the derivative of $f^{-1}$ to the derivative of $f$. *Hint:* One way to do this is to use implicit differentiation: if $y = f^{-1}(x)$, then $x = f(y)$, and take $d/dx$ of both sides.