Worksheet 10-29-15 - MAC 2311, Fall 2015 **NAME:** ____________________

1. Use l'Hopital (after an appropriate trick) to evaluate the following limits (a is a positive constant).

(a) $\lim_{x \to 0^+} x^{(\ln a)/(1+\ln x)}$

(b) $\lim_{x \to +\infty} x^{(\ln a)/(1+\ln x)}$

(c) $\lim_{x \to 0} \Big(1+(\ln a)\cdot x\Big)^{1/x}$

Of course, changing the value of the constant a, the results for your limits above change. This should convince you that 0^0, ∞^0, 1^∞ are exceptional cases for limits (or limit indeterminate forms) that could lead to any result.

2. (a) List all exceptional cases for limits.

(b) Is 0^∞ an exceptional case for limits? Briefly justify .

LECTURE: Critical points, Increasing/Decreasing, Concavity, Inflection points.

3. Sketch (if possible) the graph of a function $f(x)$ so that $f(x) > 0$, $f'(x) < 0$, $f''(x) > 0$ for all real numbers x.

4. True or False questions. In each case briefly justify your answer.

(a) If $f'(2) = 0$, $f'(x) < 0$ if $x < 2$ and $f'(x) > 0$ if $x > 2$ then f has a relative minimum at $x = 2$.

(b) If $f'(2) = 0$ then f has a relative minimum or a relative maximum at $x = 2$.

(c) If $f'(2) = 0$ and $f''(2) > 0$ then f has a relative maximum at $x = 2$.

5. For $f(x) = x^4 - 6x^2 + 5$ do the following:

(a) Find the intervals on which f is increasing; on which f is decreasing.
(b) Find the coordinates of critical points (if any) and determine whether a relative minimum, relative maximum or neither occurs there.
(c) Find the intervals on which f is concave up; on which f is concave down.
(d) Find the coordinates of all inflection points (if any) .
(e) Does the function have any asymptotes (vertical or horizontal). Justify with limits.
(f) Does the function have any symmetry? (Is it even or odd function?)
(g) Graph the function.

6. Do all the steps of the previous problem to obtain the graph of $f(x) = e^{-x^2}$.