Worksheet #15 **11-03-15** **NAME:** ____________________

LECTURE: Increasing/Decreasing. Concavity. Relative extrema. First and second derivative tests.

1. Find the critical points and characterize them as relative maxima, relative minima, or neither:
a) $f(x) = x^4 - 18x^2$ b) $f(x) = x(x-4)^3$ c) $f(x) = \frac{x^2}{x^4+16}$

2. State whether each is true or false and briefly justify your answer.
(a) If $f'(2) = 0$, $f'(x) < 0$ if $x < 2$ and $f'(x) > 0$ if $x > 2$ then f has a relative minimum at $x = 2$.

(b) If $f'(2) = 0$ then f has a relative minimum or a relative maximum at $x = 2$.

(c) If $f'(2) = 0$ and $f''(2) < 0$ then f has a relative maximum at $x = 2$.

3. Sketch (if possible) the graph of a function $f(x)$ so that $f(x) > 0$, $f'(x) < 0$, $f''(x) > 0$ for all real numbers x.

LECTURE: Graphing.

4. For $f(x) = x(x-4)^3$ do the following:

(a) Determine the domain and check if the function has any symmetry. (Is it even or odd function?)
(b) Find the derivative and find the coordinates of the critical points (if any).
(c) Use a sign chart (table) to find the intervals on which f is increasing; on which f is decreasing.
(d) Determine the type of critical points (relative minimum, relative maximum or neither).
(e) Compute f'' and find the intervals on which f is concave up; on which f is concave down.
(f) Find the coordinates of all inflection points (if any) .
(g) Does the function have any asymptotes (vertical or horizontal)? Justify with limits.
(h) Axis intercepts.
(i) Graph the function.

5. Repeat the steps of the previous problem to obtain the graph of $f(x) = e^{-x^2}$.

6. Sketch the complete graph of each of the following functions. Your work should include: the domain of the function, equations of eventual asymptotes (vertical or/and horizontal), a sign chart for the derivative and the second derivative, the location and nature of the critical points (if any), location of inflection points (if any), coordinates for the axis intercepts.

(a) $f(x) = \frac{2x+1}{x+1}$ (b) $f(x) = 1 - \frac{1}{x^2+4}$

(c) $f(x) = \frac{\ln|x|}{x}$ (d) $f(x) = x - 3x^{1/3}$