

1) Sketch the graph of $f(x) = 2 + \frac{1}{x^2-4}$ by carrying out the following steps.

- (a) Determine the domain and check if the function has any symmetry. (Is it an even or odd function or neither?)
- (b) Find the derivative and find the coordinates of the critical points (if any).
- (c) Use a sign chart (table) to find the intervals on which f is increasing; on which f is decreasing.
- (d) Determine the type of critical points (relative minimum, relative maximum or neither).
- (e) Compute f'' and find the intervals on which f is concave up; on which f is concave down.
- (f) Find the coordinates of all inflection points (if any) .
- (g) Does the function have any asymptotes (vertical or horizontal). Justify with limits.
- (h) Sketch the graph of the function, showing all the information you have gathered.

LECTURE: Absolute extrema on closed, finite interval

2) True or False. Answer and briefly justify. The justification may be just a graph.

(a) Any continuous function $f(x)$ defined on the interval $[-2, 3]$ has an absolute maximum and an absolute minimum on the interval $[-2, 3]$.

(b) There are continuous functions $f(x)$ defined on the interval $(-2, 3)$ which have neither an absolute maximum nor an absolute minimum on the interval $(-2, 3)$.

(c) Every continuous functions $f(x)$ defined on the interval $[0, +\infty)$ has an absolute maximum or an absolute minimum on the interval $[0, +\infty)$, but maybe not both.

(d) If $f(x)$ is differentiable on the interval $[0, 1]$ and $f'(x) < 0$ for all $x \in [0, 1]$, then $x = 0$ is the absolute maximum for $f(x)$ on the interval $[0, 1]$.

(e) Suppose we know that $x = 3$ is the only critical point of the function $f(x)$ on the interval $(0, +\infty)$ and we also know that $f''(x) < 0$ for all $x \in (0, +\infty)$. Then $x = 3$ must be an absolute maximum for $f(x)$ on the interval $(0, +\infty)$.

3) Find the absolute maxima and minima of the following functions on the indicated intervals:

(a) $f(x) = 8x - x^2$ on $[0, 6]$,

(b) $f(x) = e^x + e^{-x}$ on $[-2, 2]$

LECTURE: Absolute extrema on open intervals.

4) Find the absolute maxima and minima of the following functions on the indicated intervals or explain why there are none:

(a) $f(x) = \frac{1}{1-x^2}$ on the interval $(-1, 1)$,

(b) $f(x) = e^x + e^{-x}$ on $(-\infty, \infty)$

LECTURE: Applied optimization problems.

5) The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences on its other two sides. Find the dimensions of the field with the maximum area that can be enclosed using 1000 feet of fencing. Solve this problem by carrying out the following steps:

(a) Sketch the field. Label variables and indicate which ones can vary.

(b) Identify what variable you are trying to optimize.

(c) Express the variable to be optimized as a function of the variables you found in (a).

(d) Find relations among the variables from (a) and express the variable to be optimized as a function of just one of the variables from (a).

(e) Identify the domain of the one remaining variable.

(f) Find the absolute maximum of the variable to be optimized on this domain.

(g) Reread the question and be sure you have answered exactly what was asked.