

Name: _____

Panther ID: _____

Exam 3

Calculus I

Fall 2016

Important Rules:

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work may receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (10 pts) These are True or False questions. In each case, circle your answer. No justification is necessary for this problem.

(a) (2 pts) If $f'(5) = 0$ and $f''(5) < 0$ then $f(x)$ has a relative maximum at $x = 5$. **True False**

(b) (2 pts) $0^{+\infty}$ is an indeterminate form for limits. **True False**

(c) (2 pts) $\int [f(x) - 3g(x)] dx = \int f(x) dx - 3 \int g(x) dx$. **True False**

(d) (2 pts) If $f(x)$ is continuous on $(0, 4)$, then f has an absolute minimum on $(0, 4)$. **True False**

(e) (2 pts) If $f'(x) < 0$ for all $x \in [0, 4]$, then, on the interval $[0, 4]$, $f(x)$ has an absolute minimum at $x = 4$. **True False**

2. (16 pts) Find the following limits:

(a) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

(b) $\lim_{x \rightarrow +\infty} x^{1/x}$

3. (10 pts) (a) (6 points) Find the local linear approximation of the function $f(x) = \sqrt[3]{x}$ at $x_0 = 8$.

(b) (4 pts) Use the result of part (a) to approximate $\sqrt[3]{7.94}$.

4. (14 pts) A rectangular area of 1200 ft^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$3 per foot. Find the dimensions of the rectangle of least cost.

5. (12 pts) Consider $f(x) = x + \frac{1}{x}$.

(a) Find the absolute maximum and minimum values of f , if any, on $[\frac{1}{2}, 3]$.

(b) Find the absolute maximum and minimum values of f , if any, on $(0, +\infty)$.

6. (24 pts) Compute the following integrals (6 pts each):

(a) $\int \left(3^x + \frac{1}{1+x^2} - 2 \right) dx$

(b) $\int \frac{2x + \sqrt{x}}{x^2} dx$

(c) $\int \sin(3x)e^{\cos 3x} dx$

(d) $\int \frac{1}{x\sqrt{1 - (\ln x)^2}} dx$

7. (24 pts) Consider the function $f(x) = \frac{x^2}{x^2-1}$.

(a) Find vertical and horizontal asymptotes of f , if any;

(b) Use a sign chart to find the intervals on which f is increasing; on which f is decreasing. Find critical points, if any, and determine the type of critical points (relative minimum, relative maximum or neither).

(c) Compute f'' and find the intervals on which f is concave up; on which f is concave down. Find the coordinates of all inflection points (if any).

(d) Sketch the graph of the function, showing all the information you have gathered in (a)-(c).