Quiz 1 MAC-2311 Spring 2016 **Name:** Solution Key/Grading Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. Compute each of the following limits. If the limit does not exist or is infinite, specify so (2.5pts each).

(a) $\lim_{x\to 0} \dfrac{x^3-4x}{x^2-5x+6}$ $= \dfrac{0}{6} = 0$

2pts 0.5pts

(b) $\lim_{x\to 2} \dfrac{x^3-4x}{x^2-5x+6}$ $= \dfrac{0}{0}$ ← (0.5pts)

$= \lim_{x\to 2} \dfrac{x(x+2)(x-2)}{(x-3)(x-2)} = \dfrac{2\cdot 4}{(-1)} = -8$

1.5pts ← factoring 0.5pts

(c) $\lim_{x\to 3^-} \dfrac{x^3-4x}{x^2-5x+6}$ $=$

$= \lim_{x\to 3^-} \dfrac{x(x+2)\cdot(x-2)}{(x-3)(x-2)} = \dfrac{3\cdot 5}{0^-} = -\infty$

(0.5pts) 1pt 1pt

(but can be done without this step) (-0.5pts for incorrect sign)

(d) $\lim_{x\to -\infty} \dfrac{x^3-4x}{x^2-5x+6}$ $=$ Limit Rule (0.5pts ← mention limit rule, (1pt))

(1pt)

$= \lim_{x\to -\infty} \dfrac{x^3}{x^2} = \lim_{x\to -\infty} x = -\infty$ (1pt)

or with full justification

$\lim_{x\to -\infty} \dfrac{x^3\left(1-\frac{4}{x^2}\right)}{x^2\left(1-\frac{5}{x}+\frac{6}{x^2}\right)} = \dfrac{(-\infty)\cdot 1}{1} = -\infty$

(1.5 pts) (1pt)

2. (Bonus 2 pts) Does the function $f(x) = \dfrac{x^3-4x}{x^2-5x+6}$ have any vertical or horizontal asymptotes?

Answer and briefly justify. Note that in Pb. 1 you computed some limits of this function.

(1pt) $x=3$ is a vertical asymptote as limit (c) shows.

$x=2$ is not a vertical asymptote since $\lim_{x\to 2} f(x) = -8$, so the graph will just have a circle (removed point) at $(2,-8)$

(1pt) There is no horizontal asymptote since both $\lim_{x\to -\infty} f(x)$ and $\lim_{x\to +\infty} f(x)$ are infinite

$-\infty$ ← shown in (d) $+\infty$ ← as in (d)