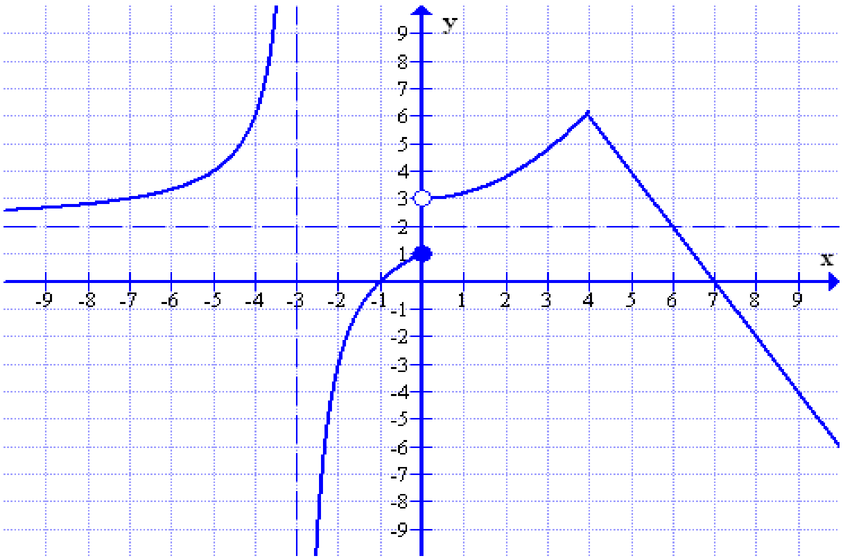
Name:_______________________________ **PID:** __________________________

Spring 2017 -- MAC 2311- Exam 1 – Version B

There are 8 problems for a total of 110 points. **Show your work**; an answer alone, even correct, may get no credit. An illegible solution will not be graded. **Calculators are not allowed**.

Problem 1. (12pts) The graph of a function *f* is given below. Use the graph to answer the questions that follow.



(i) (7 pts) Find the following limits. You don't have to show any work for these, but specify if a limit is infinite or does not exist.

$\lim_{x \to -3^-} f(x) =$ $\lim_{x \to -3^+} f(x) =$ $\lim_{x \to -3} f(x) =$

$\lim_{x \to 4} f(x) =$ $\lim_{x \to 0} f(x) =$

$\lim_{x \to -\infty} f(x) =$ $\lim_{x \to +\infty} f(x) =$

(ii) (3 pts) Is *f* continuous everywhere? If not, give *x* value(s) at which *f* has a discontinuity. Specify if any of the discontinuities is removable.

(iii) (2 pts) Identify any point(s) *x*, where the function is continuous, but it is not differentiable. Specify if there is no such point *x*.

Problem 2. (30 pts) Find the following limits. Specify if a limit is infinite or does not exist. Show all work and explain clearly (5 pts each).

a) $\lim\limits_{x \to 1^+} \dfrac{5x-1}{1-x} =$

b) $\lim\limits_{x \to -2} \dfrac{2x^2-8}{x^2-x-6} =$

c) $\lim\limits_{x \to 6} \dfrac{\sqrt{2x+4}-4}{x-6} =$

d) $\lim_{x \to +\infty} \cos\left(\frac{2\pi x}{3x+1}\right) =$

e) $\lim_{x \to 0} \frac{x \sin(6x)}{\tan^2(3x)} =$

f) $\lim_{t \to -\infty} \frac{\sqrt{3t^2 - t + 1}}{2t} =$

Problem 3. (12 pts) These are true or false questions. Answer (1pt) and give brief justification (2pts). Graph can serve as a justification.

(a) If a function *f* is continuous at $x=0$, then *f* is differentiable at $x=0$. **True** **False**

Justification:

(b) If $f(x)$, $g(x)$ are continuous everywhere then $\frac{f(x)}{g(x)}$ is continuous everywhere. **True** **False**

Justification:

(c) A function can never cross its horizontal asymptote. **True** **False**

Justification:

(d) If a function satisfies $|f(x) - 7| \leq 5|x-3|$ for all real numbers x, then $\lim_{x \to 3} f(x) = 7$. **True** **False**

Justification:

Problem 4. (10 pts) Use the limit definition of the derivative to compute $f'(x)$ for $f(x) = 1/x$.

Problem 5. (12 pts) A stone is thrown straight up from the ground. Its position *s(t)* in feet above the ground after *t* seconds is given by $s(t) = 48t - 16t^2$.

(a) (2 pts) When does the stone land back on the ground?

(b) (4 pts) Find the average velocity of the stone in the time interval [1,3] seconds.

(c)(6 pts) Use a limit to find the instantaneous velocity of the stone at *t*=1 second.

Problem 6. (12 pts) (a) (2pts) Write the definition for a function *f(x)* to be continuous at *x=a*.

(b) (5pts) Use this definition to determine whether or not the following function is continuous at *x=0*.

$$f(x) = \begin{cases} \dfrac{2x^2+5}{x^2+1} & ,if\ \ x \le 0 \\ \dfrac{\sin(3x)}{x} & ,if\ \ x > 0 \end{cases}$$

(b) (5pts) List all asymptotes, vertical or horizontal (if any), of the function *f(x)* from part (b). Justify your answer with limits.

Problem 7. (12 pts**) (**a) (6 pts) Use the Intermediate Value Theorem to show that the equation $x^3 - 6x + 2 = 0$ has three distinct real roots. Explain thoroughly.

(b) (6 pts) Use the method of bisection to approximate one of the roots of the equation $x^3 - 6x + 2 = 0$ to within 0.25. Explain thoroughly.

Problem 8. (10 pts) Choose ONE of the following. Only ONE will be graded.

(A) Use geometry to prove the inequality $\sin x \leq x \leq \tan x$ for any $\in \left[0, \frac{\pi}{2}\right)$.

(B) Write the general (ε,δ) definition for
$\lim_{x \to a} f(x) = L$

(C)
and then use this definition to prove that $\lim_{x \to 5} (20x - 3) = 97$

.

(D)