

NAME: \_\_\_\_\_

Panther ID: \_\_\_\_\_

Exam 2 - MAC 2311

Spring 2017

**Important Rules:**

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (30 pts) Find  $dy/dx$ . Simplify when possible (6 pts each):

(a)  $y = \frac{x^5}{5} - \frac{2}{\sqrt{x}} + 3\pi^4$

(b)  $y = \sec(\tan(2x))$

(c)  $y = x^3 e^{-2x}$

(d)  $y = \arcsin(2^x)$

(e)  $y = (x^2 + 1)^{\sin x}$

**2.** (10 pts) For the function  $f(x)$  it is known that  $f(1) = 2$  and  $f'(1) = -5$ .

Given that  $g(x) = [f(x)]^3$ , find

(a) (3 pts)  $g(1)$

(b) (7 pts)  $g'(1)$

3. (12 points) These are True or False questions. Circle your answer AND give a brief justification.

(a) If  $y = e^{kx}$ , where  $k$  is a constant, then  $dy/dx = ky$ .    **True**   **False**

**Justification:**

(b) If  $g(x) = f(x) \sin x$  then  $g'(x) = f'(x) \cos x$ .    **True**   **False**

**Justification:**

(c)  $\left((\tan x)^{-1}\right)' = \frac{1}{1+x^2}$     **True**   **False**

**Justification:**

(d) If  $p(x)$  is a polynomial of degree 10, then its eleventh derivative,  $p^{(11)}(x) = 0$ .    **True**   **False**

**Justification:**

4. (12 pts) Show that  $y = x \sin(5x)$  is a solution of a differential equation of the form  $y'' + 25y = a \cos(5x)$ , for a certain constant  $a$  that you should determine.

5. (12 pts) Find the equation of the tangent line to the curve  $3x^3 - 2xy^2 = 2y^3 - 1$  at the point  $(1, 1)$ .

6. (12 pts) Recall that the volume  $V$  of a circular cylinder having radius of the base  $r$  and height  $h$  is given by  $V = \pi r^2 h$ . Suppose we have a cylinder for which both dimensions  $r$  and  $h$  vary with time  $t$ .

(a) (6 pts) How are  $dV/dt$ ,  $dr/dt$  and  $dh/dt$  related?

(b) (6 pts) At a certain instant, the height is 8 cm and is increasing at 0.2 cm/s, while the radius is 10 cm and is decreasing at 0.1 cm/s. How fast is the volume changing at that instant? Give units to your answer. Is the volume increasing or decreasing at that instant?

7. (12 pts) Given the parametric curve  $x = 3 \cos t$ ,  $y = \sin t$ ,  $0 \leq t \leq 2\pi$ :

(a) (6 pts) Sketch the curve in the  $xy$  plane, marking the coordinates of axis intercepts and indicating orientation.

(b) (6 pts) Find the coordinates of a point on the curve (if any) with the property that the tangent line to the curve at that point is parallel to the line  $y = -\sqrt{3}x$ .

**8.** (10 pts) Choose ONE:

- (a) State and prove the quotient rule. (You can use product rule, chain rule, or logarithmic differentiation.)
- (b) Find, with proof, the formula for  $(\arccos x)'$ .