

1. True or False. Answer and briefly justify. The justification may be just a graph.

(a) Any continuous function $f(x)$ defined on the interval $[-2, 3]$ has an absolute maximum and an absolute minimum on the interval $[-2, 3]$.

(b) There are continuous functions $f(x)$ defined on the interval $(-2, 3)$ which have neither an absolute maximum nor an absolute minimum on the interval $(-2, 3)$.

(c) Every continuous functions $f(x)$ defined on the interval $[0, +\infty)$ has an absolute maximum or an absolute minimum on the interval $[0, +\infty)$.

(d) If $f(x)$ is differentiable on the interval $[0, 1]$ and $f'(x) < 0$ for all $x \in [0, 1]$, then $x = 0$ is the absolute maximum for $f(x)$ on the interval $[0, 1]$.

(e) Suppose we know that $x = 3$ is the only critical point of the function $f(x)$ on the interval $(0, +\infty)$ and we also know that $f''(x) < 0$ for all $x \in (0, +\infty)$. Then $x = 3$ must be an absolute maximum for $f(x)$ on the interval $(0, +\infty)$.

2. Find the absolute maxima and minima of the following functions on the indicated intervals or explain why there are none:

(a) $f(x) = x^3 - 9x + 1$ on $[-1, 3]$,

(b) $f(x) = x + \frac{1}{x}$ on $[1, 3]$

(c) $f(x) = (x^2 + x)^{2/3}$ on $[-2, 3]$

3. Find the absolute maxima and minima of the following functions on the indicated intervals or explain why there are none:

(a) $f(x) = x^3 - 9x + 1$ on $(-1, 3)$,

(b) $f(x) = x + \frac{1}{x}$ on $(0, +\infty)$

(c) $f(x) = (x^2 + x)^{2/3}$ on $(-\infty, +\infty)$

4. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences on its other two sides. Find the dimensions of the field with the maximum area that can be enclosed using 1000 feet of fencing.