Name: ______________________ **Panther ID:** ____________

Exam 1 - MAC2311 - SummerB 2017

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (14 pts) These are True or False questions. No justification required. No partial credit. 2 points each.

(i) If $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$ then $f(x)$ is continuous at $x = 3$. **True** **False**

(ii) $\lim_{x\to -\infty}(x^3+100x^2) = -\infty$ **True** **False**

(iii) For all $x \neq 0$, $\dfrac{\tan x}{x} = 1$ **True** **False**

(iv) The function $f(x) = \cot x$ is defined and is continuous for all real numbers x. **True** **False**

(v) If $\lim_{x\to a} f(x) = 5$ and $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ D.N.E. **True** **False**

(vi) $0 \cdot \infty$ is a limit indeterminate form (or exceptional case). **True** **False**

(vii) If $f(x)$ is a continuous function for all real numbers x, then f has no vertical asymptotes. **True** **False**

2. (12 pts) A robot moves in the positive direction along a straight line so that after t minutes its distance is $s = 2t^3$ feet from the origin.

(a) (5 pts) Find the average velocity of the robot in the time interval $1 \leq t \leq 3$ seconds. Give units to your answer.

(b) (7 pts) Use limits to find the instantaneous velocity of the robot when $t = 3$ seconds. Give units to your answer.

3. (8 pts) Given the function $g(x) = \begin{cases} kx^2 + 1 & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ 2x + k & \text{if } x > 2 \end{cases}$

find, if possible, a value of the constant k which will make $g(x)$ continuous everywhere. Justify your answer.

4. (30 pts) Find the following limits. If the limit is infinite or does not exist, specify so (5pts each).

(a) $\displaystyle\lim_{x \to -2} \frac{x^2 - 2x - 8}{8 - 2x^2}$

(b) $\displaystyle\lim_{x \to +\infty} \frac{x^2 - 2x - 8}{8 - 2x^2}$

(c) $\displaystyle\lim_{x \to 3} \frac{|x - 3|}{x^2 - 6x + 9}$

(d) $\displaystyle\lim_{x \to -2^-} \frac{x - 1}{x + 2}$

(e) $\lim_{x\to 0} \dfrac{\tan(7x)}{\sin(2x)+\sin(3x)}$

(f) $\lim_{x\to +\infty} (\sqrt{x^2+5x} - x)$

5. (10 pts) Use the Intermediate Value Theorem to show that the equation $x^4 + 3x - 1 = 0$ has at least two real solutions and locate each solution in an interval of length 0.5. Justify carefully.

6. (12 pts) Sketch the graph of ONE function $f(x)$ satisfying ALL of the following conditions.

(i) The function is defined for all real numbers;

(ii) The function is continuous everywhere except $x = 0$ and $x = 3$;

(iii) $\lim_{x \to 0^-} f(x) = -3$, $f(0) = -3$, $\lim_{x \to 0^+} f(x) = 4$;

(iv) $\lim_{x \to 3^-} f(x) = +\infty$, $f(3) = 0$, $\lim_{x \to 3^+} f(x) = -\infty$;

(v) $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to +\infty} f(x) = 2$.

7. (a) (3 pts) Write the general (ϵ, δ) definition for $\lim_{x \to a} f(x) = L$.

Choose ONE of the parts (b) and (c). Only ONE will receive credit. Note the different point values.

(b) (7 pts) Use the (ϵ, δ) definition to prove $\lim_{x \to 3}(5x-7) = 8$.

(c) (12 pts) Use the (ϵ, δ) definition to prove $\lim_{x \to 3}(2x^2+1) = 19$.

9. (10 pts) Choose ONE of the following:

(a) Assuming the inequality $\sin x \leq x \leq \tan x$ for any $x \in [0, \pi/2)$, prove that $\lim_{x \to 0^+} \frac{\sin(x)}{x} = 1$.

(b) State and prove the Junk Rule Theorem for rational functions.