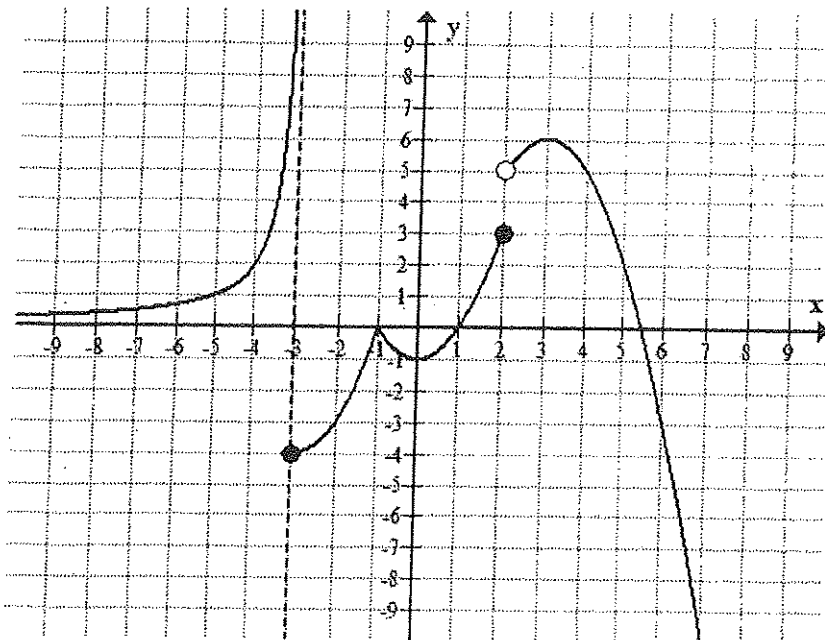


**Problem 1** The graph of a function  $f$  is given below. Use the graph to find the limits below. Specify if a limit does not exist or is infinite.



$$\lim_{x \rightarrow -3^-} f(x) =$$

$$\lim_{x \rightarrow -3^+} f(x) =$$

$$\lim_{x \rightarrow -3} f(x) =$$

$$\lim_{x \rightarrow 3} f(x) =$$

$$\lim_{x \rightarrow 2^-} f(x) =$$

$$\lim_{x \rightarrow 2^+} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

$$\lim_{x \rightarrow +\infty} f(x) =$$

**Problem 2** Sketch the graph of a function  $y = f(x)$  which satisfies all of the following conditions:

(i) the domain of  $f$  is  $(0, +\infty)$ ;

(ii)  $f(2) = f(4) = 0$ ;

(iii)  $\lim_{x \rightarrow 0^+} f(x) = -\infty$ ;

(iv)  $\lim_{x \rightarrow 2} f(x) = +\infty$ ;

(v)  $\lim_{x \rightarrow 4^-} f(x) = 0$  and

$\lim_{x \rightarrow 4^+} f(x) = 1$ ;

(vi)  $\lim_{x \rightarrow +\infty} f(x) = 3$ .

3. Find the following limits, provided they exist:

$$(a) \lim_{x \rightarrow 0} \frac{x^2 - 3x}{x^2 - 4x + 3}$$

$$(b) \lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 4x + 3}$$

$$(c) \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2 - x}$$

$$(d) \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{|2 - x|}$$

4. Find the following limits, provided they exist:

$$(a) \lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

$$(b) \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$$

$$(c) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^3 + 6x^2 - 8x + 1}$$

$$(d) \lim_{x \rightarrow 2} \frac{8 - x^3}{x^3 - 5x + 2}$$