Worksheet 06/23 - MAC 2311, Summer B 2017 **Group number:** ______________

1. Compute each of the following limits:

(a) $\lim_{x \to 0} \frac{\sin(5x)}{x} =$ $\lim_{x \to 0} \frac{\sin(ax)}{x} =$

(b) $\lim_{x \to 0} \frac{\tan(3x)}{x} =$ $\lim_{x \to 0} \frac{\tan(bx)}{x} =$

(c) $\lim_{x \to 0} \frac{1 - \cos(x)}{x}$

(d) $\lim_{x \to 0} \frac{\tan^2(3x)}{x \sin(5x)}$

(e) $\lim_{x \to 0} \frac{\sin(3x^2) + x^2}{\sin^2(3x)}$

(f) $\lim_{x \to +\infty} x \tan(3/x)$ Hint: Use the substitution technique.

(g) $\lim_{x \to +\infty} \frac{\sin(5x)}{x}$ Hint: Be careful! Here x does not go to zero!

2. (a) Write the ϵ-δ definition of $\lim_{x \to a} f(x) = L$

(b) Use the definition of limit to prove that $\lim_{x \to -3} (2x - 7) = -13$.

3. True or false. Answer and justify your answer.

(a) If $\lim_{x \to 2} f(x) = f(2) = 5$, then $4.9 < f(x) < 5.1$ for all x in a small enough interval around 2.

(b) If $\lim_{x \to 2} f(x) = f(2) = 5$, then $f(x) \neq 4.99$ for all x in a small enough interval around 2.

4. (Challenge problem) Use the definition of limit to prove that $\lim_{x \to 3} \frac{1}{x} = \frac{1}{3}$.