Worksheet 06/26 Calculus I Summer B 2017 **Group nr:** ______________

1. Consider the function $f(x) = \frac{3-x}{x^2-9}$.

(a) Determine the points of discontinuity for $f(x)$.

(b) Use limits to understand the behavior of the function near the points of discontinuity. Are any of these removable discontinuities?

(c) Does this function have vertical asymptotes? Briefly justify your answer.

(d) Does this function have horizontal asymptotes? Justify your answer with limits.

(c) Graph this function.

2. (a) Find, if possible, a value for the constant $k \geq 0$ which will make the function $g(x)$ continuous at $x = 0$.

$$g(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x < 0 \\ 1 + \sin(3x) & \text{if } x \geq 0 \end{cases},$$

(b) If there was a constant k satisfying part (a), for this value of k is the function $g(x)$ continuous everywhere? Briefly justify.

3. (a) Use IVT to show that the equation $x^3 = 3x - 1$ has a solution in the interval $[0, 1]$.

(b) Approximate the solution in part (a) with an accuracy of 0.25; that is find an interval of length 1/4 which contains the solution.

(c) Use again IVT to show that the equation $x^3 = 3x - 1$ has three real solutions and find intervals of length 1 containing each solution.

4. (a) Use the ϵ-δ definition of limit to prove that $\lim_{x \to 5}(2x+3) = 13$.

Challenge: (b) Use the ϵ-δ definition of limit to prove that $\lim_{x \to 5} \frac{1}{2x+3} = \frac{1}{13}$.

5. True or False questions. Answer and briefly justify your answer in each case.

(i) If $f(x)$ is a continuous function and $\lim_{x \to 3} f(x) = 4$ then $f(3) = 4$ **True** **False**

(ii) $\lim_{x \to \infty} \cos\left(\frac{\pi x^2}{2x^2+1}\right) = 0$ **True** **False**

(iii) The function $f(x) = \sec x$ is defined and is continuous for all real numbers x. **True** **False**

(iv) If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = 0$ **True** **False**

(v) If $|f(x) + 7| \leq 3|x + 2|$ for all real x, then $\lim_{x \to -2} f(x) = -7$ **True** **False**

(vi) If $f(x)$ is continuous at $x = 2$ and $f(2) = 5$, then for x sufficiently close to 2, $f(x) > 4.95$. **True** **False**