

~~Panther ID:~~ _____

~~NAME:~~ Group # _____

MAC 2311: Worksheet ^{07/03}~~09/29/2015~~ (Rules of Differentiation II and Derivatives of Trigonometric Functions)

LECTURE INTRO: Compute derivative of $\sin(x)$ and give the one for $\cos(x)$. (Mention that x is in radians.) Derive the product law and give the quotient law.

1) Compute the following derivatives:

a) $\frac{d}{dx}(x^3 \sin(x))$

b) $\frac{d}{dx}(3 \sin^2(x))$

c) $\frac{d}{dx}(\sqrt{x}(x^2 - 7x))$

d) $\frac{d}{dx} \frac{x^2+1}{\sqrt{x+3}}$

2) Using your knowledge of the derivatives of $\sin(x)$ and $\cos(x)$ and of the product and quotient laws, compute the following derivatives:

a) $\frac{d}{dx} \tan(x)$

b) $\frac{d}{dx} \cot(x)$

c) $\frac{d}{dx} \sec(x)$

d) $\frac{d}{dx} \csc(x)$

3) Compute the following derivatives:

a) $\frac{d}{dx} \left(5 + \frac{1}{\tan(x)} \right)$

b) $\frac{d}{d\theta} \left(\frac{\sec(\theta)}{1 + \sec(\theta)} \right)$

c) $\frac{d}{dt} ((\sin(t) + \cos(t)) \csc(t))$

4) Use this table:

x	2	3	4	5	6
$f(x)$	2	3	4	3	2
$g(x)$	7	3	-1	2	1
$f'(x)$	5	7	-1	-2	10
$g'(x)$	3	-2	1	8	-1

to calculate the value of these derivatives:

1. $\frac{d}{dx} \left[x^2 f(x) \right] \Big|_{x=2}$

3. $\frac{d}{dx} \left[f^2(x) g(x) \right] \Big|_{x=2}$

2. $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] \Big|_{x=4}$

4. $\frac{d}{dx} \left[3g(x) - 4 \frac{f(x)}{g(x)} \right] \Big|_{x=6}$

5) Imagine that a mass is attached to a wall by a spring. The mass is pushed so that it moves towards and away from the wall and the function that describes the distance of the mass (in meters) over time (seconds) is given by

$$s(t) = \frac{1}{t} \sin t$$

for $t \in [0, 2\pi]$. Determine the function that describes the velocity of the mass. What are the units of this function?

6) A telescope on the ground is tracking a rocket which is rising vertically from a launchpad. The telescope is 5 kilometers from the launchpad.

a) Sketch a triangle whose vertices are given by (i) the telescope, (ii) the launchpad, and (iii) the rocket. Do not assume that the rocket is on the launchpad.

b) Identify which angle in this triangle is the right angle.

c) Write the altitude of the rocket (i.e. height over the launchpad) as a function of the angle the telescope makes with the ground.

d) Compute the rate of change of this altitude with respect to the angle.

7) In this exercise, we derive the “quotient rule” giving the derivative of (f/g) :

a) Use the limit definition of the derivative to compute

$$\frac{d}{dx} \frac{1}{x} =$$

b) Let $g(x)$ be a function and assume $g(x_0) \neq 0$ and $g'(x_0)$ exists. Write down the limit defining the derivative

$$\left(\frac{1}{g(x)} \right)'$$

at x_0 and compute this limit, using the algebraic method from (a).

c) Use the formula for $(1/g(x))'$ from (b) and the product rule to compute:

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{d}{dx} \left(f(x) \frac{1}{g(x)} \right) =$$