Worksheet 07/07 Calculus I Summer B 2017 **Group nr:** _____

1. Show that the function $y = \sin(x^2)$ satisfies the differential equation $y'' - \frac{y'}{x} + ax^2y = 0$, for a certain constant a that you should determine.

2. For each of the following implicitly defined functions, find $\frac{dy}{dx}$:

(a) $y^4 - 3y^3 - x = 3$

(b) $\cos(xy) = x - y$

3. Consider the function implicitly defined by $y^4 = x + y$.

a) Find an expression for the derivative $\frac{dy}{dx}$.

b) Find the equation of the line tangent to this function at the point (0,1).

c) Find where the tangent line is vertical.

4. Find, with proof, the formulae for $(\arctan(x))'$ and $(\text{arccot(x)})'$.

5. Compute the derivative of each of the following functions:

a) $y = \arctan(\sin(x))$

b) $y = \cos(x)\tan^{-1}(2x)$

c) $y = \sin^{-1}(\cos(3x))$

d) $y = \frac{x^3+7}{\arctan(x^2)}$

6. Show that the function $f(x) = 2x^3 + 6x - 5$ is one to one and then find $(f^{-1}(3))'$. Note that $f(1) = 3$.

7. A ten-foot long, straight plank is leaning against a vertical wall when it begins to slip. Suppose the base of the plank is moving away from the wall at 2ft./s.. How fast is the top of the plank moving down the wall when the top is 6ft above the ground?