Worksheet 07/10 Calculus I Summer B 2017 **Group nr:** ____________

1. True or False. Answer and briefly justify. The justification may be just a graph.
(a) Any continuous function $f(x)$ defined on the interval $[-2,3]$ has an absolute maximum and an absolute minimum on the interval $[-2,3]$.

(b) There are continuous functions $f(x)$ defined on the interval $(-2,3)$ which have neither an absolute maximum nor an absolute minimum on the interval $(-2,3)$.

(c) Every continuous functions $f(x)$ defined on the interval $[0,+\infty)$ has an absolute maximum or an absolute minimum on the interval $[0,+\infty)$.

(d) If $f(x)$ is differentiable on the interval $[0,1]$ and $f'(x) < 0$ for all $x \in [0,1]$, then $x = 0$ is the absolute maximum for $f(x)$ on the interval $[0,1]$.

(e) Suppose we know that $x = 3$ is the only critical point of the function $f(x)$ on the interval $(0,+\infty)$ and we also know that $f''(x) < 0$ for all $x \in (0,+\infty)$. Then $x = 3$ must be an absolut maximum for $f(x)$ on the interval $(0,+\infty)$.

2. Find the absolute maxima and minima of the following functions on the indicated intervals or explain why there are none:

(a) $f(x) = x^3 - 9x + 1$ on $[-1,3]$,

(b) $f(x) = x + \frac{1}{x}$ on $[1,3]$

(c) $f(x) = (x^2 + x)^{2/3}$ on $[-2,3]$

(d) $f(x) = x^3 - 9x + 1$ on $(-1,3)$,

(e) $f(x) = x + \frac{1}{x}$ on $(0,+\infty)$

(f) $f(x) = (x^2 + x)^{2/3}$ on $(-\infty,+\infty)$

3. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences on its other two sides. Find the dimensions of the field with the maximum area that can be enclosed using 1000 feet of fencing.

4. You are designing a poster that will contain 50 square inches of text, which must have a 4 inch margin at the top and bottom and a 2 inch margin on the left and right sides. What overall dimensions will minimize the amount of paper that needs to be used?

5. A square-based shipping crate is being designed that must contain a volume of 16 ft^3. The material that is used for the base and the lid costs 3 dollars/ft^2, while the material used for the sides costs 2 dollars/ft^2. What are the most cost-effective dimensions of such a crate?

6. What are the dimensions of the lightest cylindrical can that can hold a given volume of V_0 cm^3? Assume the same material is used for the lid, the base and the side of the can. [Hint: the lightest can is the can with the smallest surface area.]

7. Suppose that a tour service offers the following rates:

- \$200 per person if 50 people go on the tour (this is also the minimum number of people needed to book the tour).
- For each additional person over 50, up to a maximum of 80 total people, the rate per person is reduced by \$2.

It costs \$6000 (a fixed cost) plus \$32 per person to conduct the tour. What number of people will maximize the profit per tour?