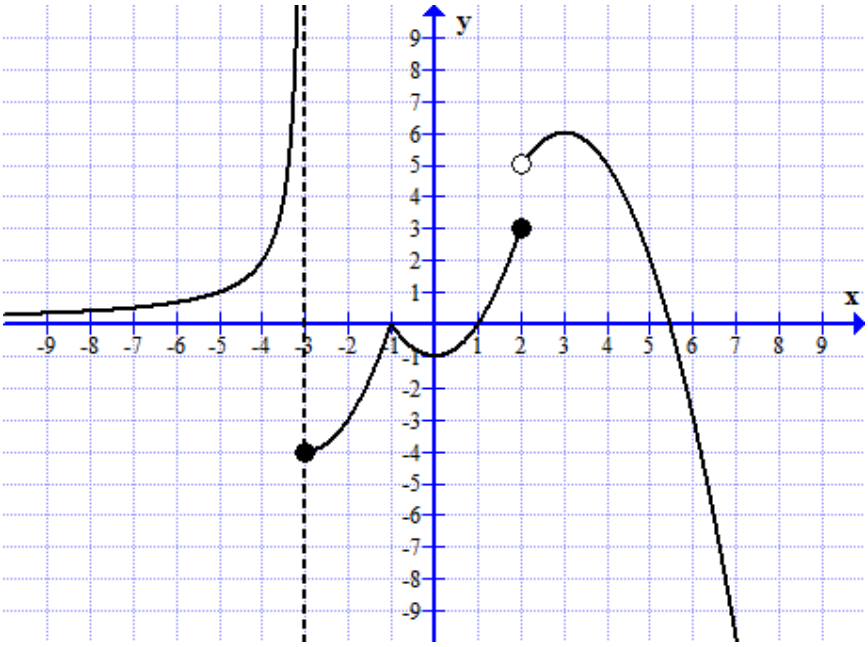
**Name:**_____ **PID:** _____

**Summer 2019 -- MAC 2311- Exam 1**

There are 8 problems for a total of 109 points. **Show your work**; an answer alone, even correct, may get no credit. An illegible solution will not be graded. **Calculators are not allowed**.

**Problem 1. (17 pts)** The graph of a function *f* is given below. Use the graph to answer the questions that follow.




(i) (7 pts) Find the following limits (you don't have to show any work here)

$\lim_{x\to -3^-} f(x) =$ $\lim_{x\to -3^+} f(x) =$ $\lim_{x\to -3} f(x) =$

$\lim_{x\to 2} f(x) =$ $\lim_{x\to 0} f(x) =$

$\lim_{x\to -\infty} f(x) =$ $\lim_{x\to +\infty} f(x) =$

(ii) (2 pts) Specify the domain of the function *f* .

(iii) (3 pts) Is *f* continuous everywhere? If not, give *x* value(s) at which *f* has a discontinuity. Specify if any of the discontinuities is removable.

(iv) (2 pts) Identify any point(s) *x*, where the function is not differentiable. Specify if there is no such point *x*.

(v) (3pts) On the same coordinate system, sketch the graph of *f'(x)*.

**Problem 2**. (30 pts) Find the following limits. If a limit is infinite or does not exist, specify so. (5 pts each).

a) $\lim_{x \to -3^+} \frac{x}{3+x}$

b) $\lim_{x \to 3} \frac{x^2 - x + 6}{9x - x^3}$

c) $\lim_{x \to +\infty} \frac{x^2 - x + 6}{9x - x^3}$

d) $\lim_{x \to +\infty} \cos(x) =$

e) $\lim_{x \to 0} \frac{\tan 2x}{\sin 5x}$

f) $\lim_{x \to +\infty} \sqrt{x^2 + 3x}$ - x

**Problem 3. (9 pts)** These are true or false questions. Answer (1pt) and give brief justification (2pts). Graph can serve as a justification.

(a) The graph of a function can never cross its horizontal asymptote. **True** **False**

**Justification:**

(b) If a function *f* is continuous at *x=0*, then f is differentiable at *x=0*. **True** **False**

**Justification:**

(c) The equation $x^3 = 3x^2 + 1$ has a solution in the interval [3,4]. **True** **False**

**Justification:**

**Problem 4.** (8 pts) Use the limit definition of the derivative to compute *f'(x)* for *f(x)* = $\sqrt{x}$ .

**Problem 5. (15 pts)** Compute *f'(x)* for the following. You don't need to use limits here. (5 points each)

a) $f(x) = x^3 e^x$

b) $f(x) = \frac{x^2-4}{3x+1}$

c) $f(x) = \pi^2 + 3\sqrt{x} - \frac{5}{x}$

**Problem 6. (12 pts)** (a) (2pts) Write the limit definition of continuity for a function *f(x)* at *x=a*.

(b) (5pts) Use this definition to determine whether or not the following function is continuous at *x=0*.

$$f(x) = \begin{cases} \dfrac{x^2+5}{2x^2+1} & ,if\ x \leq 0 \\ \dfrac{sin(3x)}{x} & ,if\ x > 0 \end{cases}$$

(c)(5pts) List all asymptotes, vertical or horizontal (if any), of the function *f(x)* from part (b). Justify your answer with limits.

**Problem 7.** (8 pts) A particle is moving on the $x$-axis. Its position (in cm) at time $t$ (in seconds) is given by $x(t)=t^3 + t - 9$. Answer the following (2 pts each)

a) Find the instantaneous velocity at time $t$=2 s.

b) Find the acceleration at $t$=2 s.

c) Find the average velocity over the time interval [0,1].

d) Find the time interval when the particle is moving to the left.

**Problem 8. (10 pts)** Choose ONE:

(A) State and prove the formula for the derivative of a product (the product rule).

(B) Prove the power rule for the case of a positive integer power.

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