Quiz 1 MAC-2281 Spring 2019 **Name:** Solution Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. (8 pts) Compute each of the following limits. If the limit does not exist or is infinite, specify so.

(a) $\lim\limits_{x\to -5^-} \dfrac{1+x}{x+5} = \dfrac{-4}{0^-} = +\infty$

(b) $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-5x+6} \overset{0/0}{=} \lim\limits_{x\to 2} \dfrac{\cancel{x-2}^{\,1}}{(\cancel{x-2})(x-3)} = \dfrac{1}{2-3}$

$\boxed{-1}$

(c) $\lim\limits_{x\to -\infty} \dfrac{3x^2-2}{x^2-7x+2} \overset{\infty/\infty}{=} *$

Junk Rule

$= \lim\limits_{x\to -\infty} \dfrac{3x^2}{x^2} = \boxed{3}$

More rigorously (but above solution OK)

$* = \lim\limits_{x\to -\infty} \dfrac{\cancel{x^2}\left(3-\frac{2}{x^2}\right)}{\cancel{x^2}\left(1-\frac{7}{x}+\frac{2}{x^2}\right)} = \dfrac{3}{1} = \boxed{3}$

(d) $\lim\limits_{x\to +\infty} \sqrt{x^2-x}-x \overset{\infty-\infty}{=} \lim\limits_{x\to +\infty} \dfrac{\left(\sqrt{x^2-x}-x\right)\left(\sqrt{x^2-x}+x\right)}{\sqrt{x^2-x}+x}$

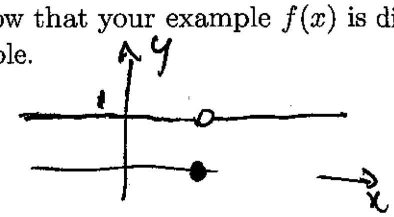
Multiply by conjugate up & down

$= \lim\limits_{x\to +\infty} \dfrac{\cancel{x^2}-x-\cancel{x^2}}{\sqrt{x^2-x}+x} \overset{\infty/\infty}{=}$

$= \lim\limits_{x\to +\infty} \dfrac{-x}{\sqrt{x^2\left(1-\frac{1}{x}\right)}+x} = \lim\limits_{x\to +\infty} \dfrac{-x}{x\sqrt{1-\frac{1}{x}}+x} =$

$= \lim\limits_{x\to +\infty} \dfrac{-\cancel{x}\,1}{\cancel{x}\left(\sqrt{1-\frac{1}{x}}+1\right)} = \dfrac{-1}{1+1} = \boxed{-\dfrac{1}{2}}$

2. (2 pts) Give an example of a function $f(x)$ that is continuous for all values of x except $x = 2$, where it has a removable discontinuity. Briefly explain how you know that your example $f(x)$ is discontinuous at $x = 2$ and how you know that the discontinuity is removable.



Exp: Let $f(x) = \begin{cases} 1 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$

$\lim\limits_{x\to 2} f(x) = 1 \neq 0 = f(2)$ so f is discontinuous at $x=2$

However the singularity is removable as $\lim\limits_{x\to 2} f(x)$ exists and is finite.

The function $\tilde{f}(x) = 1$ for all x is continuous and $\tilde{f}(x) = f(x)$ for all $x \neq 2$.

Of course, there are many other possible examples, for instance

$f(x) = \dfrac{x^2-4}{x-2}$ or $f(x) = \dfrac{x^2-5x+6}{x-2}$ or just $f(x) = \dfrac{x-2}{x-2}$