Worksheet #1 Solutions

1. average velocity = average rate of change = $\frac{s(\beta)-s(\alpha)}{\beta-\alpha}$

2. average velocity for [1, 2]:

$$= \frac{f(2)-f(1)}{2-1} = \frac{[-(2)^2+5(2)+10]-[-(1)^2+5(1)+10]}{1} = \frac{16-14}{1} = 2\ \frac{\text{units}}{\text{sec}}$$

average velocity for [1, 1.5]:

$$= \frac{f(1.5)-f(1)}{1.5-1} = \frac{[-(1.5)^2+5(1.5)+10]-14}{0.5} = \frac{15.25-14}{0.5} = 2.5\ \frac{\text{units}}{\text{sec}}$$

average velocity for [1, 1.1]:

$$= \frac{f(1.1)-f(1)}{1.1-1} = \frac{[-(1.1)^2+5(1.1)+10]-14}{0.1} = \frac{14.29-14}{0.1} = 2.9\ \frac{\text{units}}{\text{sec}}$$

slope of the tangent line (instantaneous velocity) at $t=1$.

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[-(1+h)^2+5(1+h)+10]-[-(1)^2+5(1)+10]}{h}$$

$$= \lim_{h\to 0} \frac{-(1+2h+h^2)+\cancel{5}+5h+\cancel{10}+1-\cancel{5}-\cancel{10}}{h} = \lim_{h\to 0} \frac{-\cancel{1}-2h-h^2+5h+\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{3h-h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(3-h)}{\cancel{h}} = \lim_{h\to 0} 3-h = 3-0 = 3\ \frac{\text{units}}{\text{sec}}$$

Note: As we calculated the average velocities in smaller intervals, it got closer to our slope of the tangent line.