

Worksheet - Jan. 08, 2019

## 2 Limits

### 2.1 The Idea of Limits

2.1.1 If the position of an object as it moves in one dimension is given by  $s(t)$ , what is the average velocity of this object between  $t = \alpha$  and  $t = \beta$ ?

2.1.2 Imagine that an object's position along a straight line is given by the function

$$f(t) = -t^2 + 5t + 10$$

Calculate the average velocity of this object over the intervals  $[1, 2]$ ,  $[1, 1.5]$ ,  $[1, 1.1]$ ,  $[1, 1.01]$ ,  $[1, 1.001]$  and make a table of these results. From your table, estimate the slope of the tangent line to the function at  $t = 1$ .

2.1.3 Similar to the previous problem, calculate the average velocity of an object whose position is given by

$$g(t) = 2 \sin(t)$$

over the intervals  $[\pi/2, \pi]$ ,  $[\pi/2, \pi/2 + .1]$ ,  $[\pi/2, \pi/2 + .01]$ ,  $[\pi/2, \pi/2 + .0001]$  and use these results to estimate the instantaneous velocity of the object at  $t = \pi/2$ . Why are each of the values you calculated less than the estimated instantaneous velocity at  $t = \pi/2$ ? [Hint: consider the plot of  $g(t)$ .]

2.1.4 Consider the function

$$g(x) = 2x^3$$

First, make a graph of this function. Second, using a similar method as above, estimate the slope of the line tangent to this function at  $x = 2$ .