Worksheet 01/15/2019 - **MAC 2311**

1. Find the following limits, provided they exist.

(a) $\lim_{x\to3^+}\frac{2}{3-x}$

$= \frac{2}{\text{very small } -\#} = \boxed{-\infty}$

(b) $\lim_{x\to3^-}\frac{2}{3-x}$

$= \frac{2}{\text{very small } +\#} = \boxed{\infty}$

(c) $\lim_{x\to3}\frac{2}{3-x}$

Since $\lim_{x\to3^+}\frac{2}{3-x} \neq \lim_{x\to3^-}\frac{2}{3-x}$,

$\lim_{x\to3}\frac{2}{3-x}$ $\boxed{\text{does not exist (DNE)}}$

(d) $\lim_{x\to0}\frac{3x-x^2}{x^2-4x+3}$

$= \lim_{x\to0}\frac{-x\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \lim_{x\to0} -\frac{x}{x-1}$

$= -\frac{0}{0-1} = \boxed{0}$

(e) $\lim_{x\to3}\frac{3x-x^2}{x^2-4x+3}$

$= \lim_{x\to3}\frac{-x\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \lim_{x\to3} -\frac{x}{x-1}$

$= -\frac{3}{3-1} = \boxed{-\frac{3}{2}}$

(f) $\lim_{x\to1}\frac{3x-x^2}{x^2-4x+3}$

$= \lim_{x\to1}\frac{-x\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \lim_{x\to1} -\frac{x}{x-1}$

$\lim_{x\to1^-} -\frac{x}{x-1} \stackrel{?}{=} \lim_{x\to1^+} -\frac{x}{x-1}$

$\frac{-1}{\text{very small } -\#} \stackrel{?}{=} \frac{-1}{\text{very small } +\#}$

$\infty \neq -\infty$

$\boxed{\text{does not exist}}$

(g) $\lim_{x\to2}\frac{x^2+x-6}{2-x}$

$= \lim_{x\to2}\frac{(x+3)(x-2)}{2-x} = \lim_{x\to2}\frac{(x+3)\cancel{(x-2)}}{-\cancel{(x-2)}}$

$= \lim_{x\to2} -(x+3) = -(2+3) = \boxed{-5}$

(h) $\lim_{x\to2}\frac{x^2+x-6}{|2-x|}$

$\lim_{x\to2^-}\frac{x^2+x-6}{|2-x|} \stackrel{?}{=} \lim_{x\to2^+}\frac{x^2+x-6}{|2-x|}$

$\lim_{x\to2^-}\frac{(x-2)(x+3)}{2-x} \stackrel{?}{=} \lim_{x\to2^+}\frac{(x-2)(x+3)}{-(2-x)}$

$\lim_{x\to2^-}\frac{-\cancel{(2-x)}(x+3)}{\cancel{2-x}} \stackrel{?}{=} \lim_{x\to2^+}\frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}}$

$\lim_{x\to2^-} -(x+3) \stackrel{?}{=} \lim_{x\to2^+} x+3$

$-5 \neq 5$

$\boxed{\text{does not exist}}$

2. Find the following limits, provided they exist:

(a) $\lim_{x\to-1}\frac{\sqrt{x^2+8}-3}{x+1}$

$= \lim_{x\to-1}\left(\frac{\sqrt{x^2+8}-3}{x+1}\right)\left(\frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}\right)$

$= \lim_{x\to-1}\frac{x^2+8-9}{(x+1)(\sqrt{x^2+8}+3)} = \lim_{x\to-1}\frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$

$= \lim_{x\to-1}\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(\sqrt{x^2+8}+3)} = \lim_{x\to-1}\frac{x-1}{\sqrt{x^2+8}+3}$

$= \frac{-1-1}{\sqrt{(-1)^2+8}+3} = -\frac{2}{\sqrt{1+8}+3} = \frac{-2}{\sqrt{9}+3}$

$= -\frac{2}{3+3} = -\frac{2}{6} = \boxed{-\frac{1}{3}}$

(b) $\lim_{x\to2}\frac{8-x^3}{x^3-5x+2}$

• Factoring x^3-5x+2:

$\frac{p}{q} = \frac{\pm1,\pm2}{\pm1} = \pm1,\pm2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 2 \\ & & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & \boxed{0} \end{array} \checkmark$$

$= \lim_{x\to2}\frac{-(x^3-8)}{x^3-5x+2} = \lim_{x\to2}\frac{-\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x^2+2x-1)}$

$= \lim_{x\to2}\frac{-x^2-2x-4}{x^2+2x-1} = \frac{-4-4-4}{4+4-1}$

$= \boxed{-\frac{12}{7}}$

3. Compute each of the following limits:

when $x \to 0$, $t = 5x$ approaches 0

(a) $\lim_{x\to 0} \frac{\sin(5x)}{x} = \lim_{t \to 0} \frac{\sin t}{\frac{t}{5}} = 5 \lim_{x\to 0} \frac{\sin t}{t} = \boxed{5}$

when $x \to 0$, $t = ax$ approaches 0

$\lim_{x\to 0} \frac{\sin(ax)}{x} = \lim_{t\to 0} \frac{\sin t}{\frac{t}{a}} = a \lim_{x \to 0} \frac{\sin t}{t} = a \cdot 1 = \boxed{a}$

(b) $\lim_{x\to 0} \frac{\tan(3x)}{x} = \lim_{x\to 0} \frac{\sin 3x}{x \cos 3x} = \lim_{x \to 0} \frac{\sin 3x}{x} \cdot \frac{1}{\cos 3x}$
$= (3)\left(\frac{1}{1}\right) = \boxed{3}$

$\lim_{x\to 0} \frac{\tan(bx)}{x} = \lim_{x\to 0} \frac{\sin bx}{x \cos bx} = \lim_{x\to 0} \frac{\sin bx}{x} \cdot \frac{1}{\cos bx}$
$= (b)\left(\frac{1}{1}\right) = \boxed{b}$

(c) $\lim_{x\to 0} \frac{1-\cos(x)}{x} = \lim_{x\to 0} \frac{1-\cos x}{x}\left(\frac{1+\cos x}{1+\cos x}\right) = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x}$
$= (1)\left(\frac{0}{2}\right) = \boxed{0}$

(d) $\lim_{x\to 0} \frac{\tan^2(3x)}{x\sin(5x)} = \lim_{x\to 0} \frac{\frac{\tan^2(3x)}{(3x)^2} \cdot 9\cancel{x^2}}{\cancel{x} \cdot \frac{\sin(5x)}{5x} \cdot 5\cancel{x}} = \lim_{x\to 0} \frac{\tan 3x}{3x} \cdot \frac{\tan 3x}{3x} \cdot 9 \cdot \frac{1}{5\,\frac{\sin 5x}{5x}} = \left(\frac{3}{3}\right)\left(\frac{3}{3}\right)(9)\left(\frac{1}{5}\right) = \boxed{\frac{9}{5}}$

(e) $\lim_{x\to 0} \frac{\sin(3x^2)+x^2}{\sin^2(3x)} = \lim_{x\to 0} \frac{\frac{\sin 3x^2}{3x^2} \cdot 3x^2 + x^2}{\frac{\sin^2(3x)}{(3x)^2} \cdot 9x^2} = \lim_{x\to 0} \frac{\cancel{x^2}\left(3\frac{\sin 3x^2}{3x^2} + 1\right)}{9\cancel{x^2} \cdot \frac{\sin 3x}{3x} \cdot \frac{\sin 3x}{3x}} = \frac{3+1}{9 \cdot 1 \cdot 1} = \boxed{\frac{4}{9}}$

(f) $\lim_{x\to+\infty} x\tan(3/x)$ Hint: Use the substitution technique. when $x \to \infty$, $t = \frac{3}{x}$ approaches 0

$\lim_{t\to\infty} \frac{3}{t} \tan t = \lim_{t \to \infty} 3 \cdot \frac{\tan t}{t} = 3 \cdot 1 = \boxed{3}$

(g) $\lim_{x\to+\infty} \frac{\sin(5x)}{x}$ Hint: Be careful! Here x does not go to zero!

$\sin x$ is an oscillating function whose range goes from -1 to 1. Using Squeeze Theorem,

$$-1 \le \sin 5x \le 1$$

$$-\frac{1}{x} \le \frac{\sin 5x}{x} \le \frac{1}{x}$$

$$\lim_{x\to\infty} -\frac{1}{x} \le \lim_{x\to\infty} \frac{\sin 5x}{x} \le \lim_{x\to\infty} \frac{1}{x}$$

$$0 \le \lim_{x\to\infty} \frac{\sin 5x}{x} \le 0$$

Therefore,

$$\lim_{x\to\infty} \frac{\sin 5x}{x} = \boxed{0}$$