Worksheet 01/22 Calculus I Spring 2019 **Group nr:** _____

1. Consider the function $f(x) = \frac{8-2x^2}{x^2-6x+8}$.

(a) Determine the points of discontinuity for $f(x)$.

(b) Use limits to understand the behavior of the function near the points of discontinuity. Are any of these removable discontinuities?

(c) Does this function have vertical asymptotes? Briefly justify your answer.

(d) Does this function have horizontal asymptotes? Justify your answer with limits.

2. Find, if possible, a value for the constant $k \geq 0$ which will make the function $g(x)$ continuous at $x = 0$.

$$g(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x < 0 \\ 1 + \sin(3x) & \text{if } x \geq 0 \end{cases},$$

3. True or False questions. Answer and briefly justify your answer in each case.

(i) If $f(x)$ is a continuous function and $\lim_{x\to 3} f(x) = 4$ then $f(3) = 4$ **True** **False**

Justification:

(ii) $\lim_{x\to+\infty} \cos\left(\frac{\pi x^2}{2x^2+1}\right) = 0$ **True** **False**

Justification:

(iii) The function $f(x) = \frac{x}{\sqrt{x^2+1}}$ is defined and is continuous for all real numbers x. **True** **False**

Justification:

(iv) The function $f(x) = \sec x$ is defined and is continuous for all real numbers x. **True** **False**

Justification:

(v) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$ **True** **False**

Justification:

4. The temperature in an oven is given by $T(t) = t^3 + 3t^2 + 5$, where t is given in minutes, and $T(t)$ is given in Celsius degrees. Find, to within 30 seconds, the moment when the temperature has reached one hundred degrees. (A calculator may be necessary.) What Calculus theorem are you using?