Worksheet 01/29 **MAC 2281-Spring 2019** **Group nr:** [redacted] **Names:** [redacted]

Note: Problems 1 and 2 are nearly identical to problems from section 3.1 in your textbook.
As we already covered (some) properties of the derivative from section 3.3, you are allowed to use them.

1. Find an equation for the tangent line to the curve $y = f(x) = \frac{1}{x^2}$ at $x = -1$. Then sketch the curve and tangent line together.

$f(-1) = \frac{1}{(-1)^2} = \frac{1}{1} = 1$

$f(x) = x^{-2}$

$f'(x) = -2x^{-3} = -\frac{2}{x^3}$

$f'(-1) = \frac{-2}{(-1)^3} = 2$

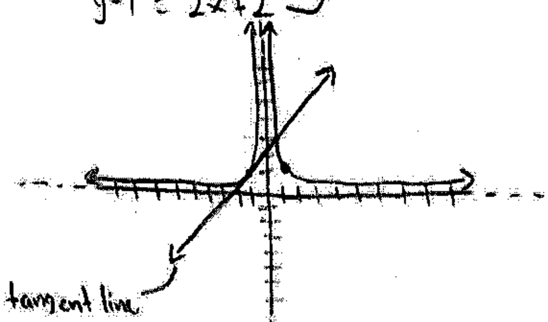
Point: $(-1, 1)$ Slope: 2

Tangent Line Equation

$y - 1 = 2(x+1)$

$y - 1 = 2x + 2$

$y = 2x + 3$



2. An object is dropped from the top of a 320-ft-high tower. Its height above the ground after t seconds is given by $s(t) = 320 - 16t^2$ feet.

(a) How fast is the object falling at $t = 2$ seconds? Give units to your answer.

$s(t) = 320 - 16t^2$

$s'(t) = -32t$

$s'(2) = -32(2) = \boxed{-64 \frac{\text{feet}}{\text{sec}}}$

(b) With what speed does the object hit the ground?

$s(t) = 0$ when the object hits the ground.

$320 - 16t^2 = 0$

$320 = 16t^2$

$20 = t^2$

$t = \sqrt{20} = \sqrt{5 \cdot 4} = 2\sqrt{5}$

$s'(2\sqrt{5}) = -32(2\sqrt{5}) = \boxed{-64\sqrt{5} \frac{\text{feet}}{\text{sec}}}$

3. The City(2,1) is connected to the rest of the XYplane world by an ancient railroad that follows the graph of the function $y = \frac{1}{4}x^2$, for $-\infty < x \le 2$. The railroad stops at the point (2,1) where the City(2,1) is located. The engineers in City(2,1) would like to extend the railroad in the wilderness territories where $x > 2$. But after the great war of 6029, they lost the technology to produce curved railroad. Now they can only produce straight railroad. They still persist and have the following plan: extend the railroad to follow the graph of a function

$$g(x) = \begin{cases} \frac{1}{4}x^2 & \text{if } x \le 2 \\ ax + b & \text{if } x > 2 \end{cases},$$

where a, b are some constants to be determined. Can you help them determine the constants a, b so that the train shifts smoothly at the point (2,1) to the new piece of railroad they want to build?

Find a and b so $\lim_{x\to 2} g(x)$ and $\lim_{x\to 2} g'(x)$ both exist

This has to do with differentiability of g(x) at x=2

$$g'(x) = \begin{cases} \frac{1}{2}x & \text{if } x < 2 \\ a & \text{if } x > 2 \end{cases}$$

$$\lim_{x\to 2^-} g'(x) = \lim_{x\to 2^+} g'(x)$$

$$\lim_{x\to 2^-} \frac{1}{2}x = \lim_{x\to 2^+} a$$

$$\frac{1}{2}(2) = a$$

$$\boxed{a = 1}$$

We'd like the slope of the parabola at (1,2) to be the same as the slope of the line.

This has to do with the continuity of g(x) at x=2

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^+} g(x)$$

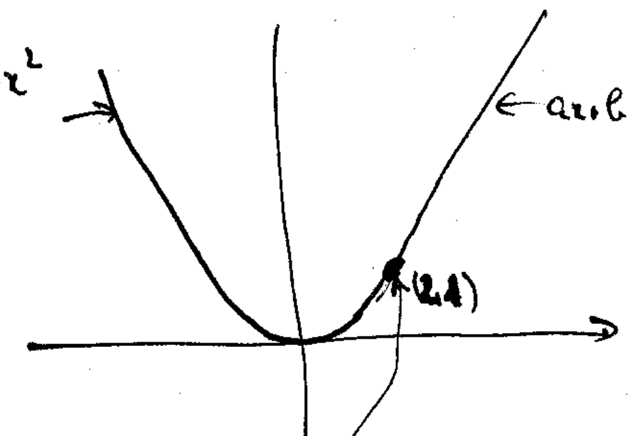
$$\lim_{x\to 2^-} \frac{1}{4}x^2 = \lim_{x\to 2^+} ax + b$$

$$\frac{1}{4}(2)^2 = 2a + b$$

$$\frac{1}{4}(4) = 2(1) + b$$

$$1 = 2 + b$$

$$\boxed{b = -1}$$



Then we would like the line $y = ax+b$ to contain the point (2,1)

~~City~~ g(x) should be continuous and differentiable at x=2